Faculdade de Engenharia da Universidade do Porto

DEVELOPMENT OF A WEB APPLICATION FOR THE DESIGN OF ADHESIVE JOINTS

A dissertation submitted in partial fulfilment
of the requirements for the degree of
Master of Mechanical Engineering

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Abstract

The design of adhesive joints is a broad topic in respect to how we predict and analyse the phenomena occurring in the adhesive connection. There are several joint types, and for each joint type there are many analytical methods whose complexity range from simple formulas to systems of differential equations that describe various aspects of the joint connection behaviour. The work that needs to be done by the engineer designing an adhesive joint is therefore dependent of the method considered, and for more complex methods the steps that need to be performed are often long and computer intensive.

The presented application tries to automate and simplify the design process so that the engineer does not have to take the time to implement all the method's equations and retrieve the necessary charts and values. A goal was set to create a user friendly interface that encompasses several analytical methods for several types of joints, where the user should need minimal effort to obtain the desired data. The decision was made to create a web interface for this task, which brings several advantages conventional software does not have, mainly the fact that the engineer could use the web interface from any place or device with an internet connection. Making the main goal the implementation of several analytical methods and creating useful output from them, other goals were set to add more value to the web application. Some of them are: the ability to share the analysis with coworkers, the ability to save joint data for future use, export utility to transfer data to a spreadsheet or other software, printing and e-mailing of results, ability to calculate the problem at hand with various analytical methods simultaneously.

It is believed that this product will greatly reduce the time taken by an engineer to design a joint connection, eliminate the chance of error and the need to recheck calculated values, and overall improving the quality and precision of the desired results.

Resumo

O projecto de juntas adesivas é um tópico extenso no que respeita à forma como prevemos e analisamos os fenómenos que ocorrem na ligação adesiva. Existem diversos tipos de juntas, e para cada tipo diversos métodos analíticos cuja complexidade varia desde simples fórmulas a sistemas de equações diferenciais que determinam vários aspectos do comportamento da ligação adesiva. O trabalho que o engenheiro precisa de efectuar para projectar a ligação adesiva para determinar as características da ligação é então dependente do método analítico considerado, e para métodos mais complexos os passos que necessitam de ser efectuados são frequentemente longos e com exigências computacionais elevadas.

A aplicação aqui apresentada tenta automatizar e simplificar o projecto da ligação adesiva tal que o engenheiro não tenha de dispender tempo para implementar todas as equações do método escolhido e retirar os gráficos e valores desejados. Foi estabelecido o objectivo de criar uma interface acessível e de fácil uso que implemente vários métodos analíticos para vários tipos de juntas adesivas, onde o utilizador pode obter os resultados desejados com a menor dificuldade possível. Tomou-se a decisão de criar um interface web para esta tarefa, trazendo este várias vantagens que um programa de software convencional não proporcionaria, principalmente o facto do utilizador poder usar o interface web de qualquer lugar ou qualquer aparelho com uma ligação à internet. Sendo o principal objectivo a implementação de vários métodos analíticos e apresentar os resultados de maneira intuitiva, outras funcionalidades secundárias foram pensadas: opção de partilhar resultados com colegas, capacidade de guardar a análise para uso futuro, funcionalidade para exportar os resultados para ficheiros (Excel, PDF, entre outros), impressão de gráficos e envio de resultados por e-mail, e ainda a possibilidade de calcular o mesmo problema com vários métodos analíticos simultaneamente.

Este produto irá reduzir significativamente o tempo que um engenheiro demora a projectar uma junta adesiva, eliminar a possibilidade de erros de cálculos e necessidade de verificar equações e valores, e no geral aumentar a qualidade e precisão dos resultados desejados.

Acknowledgments

I would like to thank:

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- My girlfriend, Cláudia, for her support and constructive criticism;
- My parents, without who I would not be here and this thesis would not be possible;
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List of Acronyms and Symbols

Acronyms

CAS	Computer Algebra System
DLJ	Double Lap Joint
DOF	Degree of Freedom
FEA	Finite Element Analysis
FEM	Finite Element Method
OS	Operating System
SLJ	Single Lap Joint

Symbols

b	Joint Width
c	Half of Overlap Length
E	Adherend Young's Modulus
E_a	Adhesive Young's Modulus
G	Adhesive Shear Modulus
h	Bond Thickness
k	Bending Moment Factor
k'	Transverse Force Factor
l	Overlap Length
M	Bending Moment
P	Applied Load
$\overline{P}$	Applied Load per Unit Width

t	Adherend Thickness
t_a	Adhesive Thickness
t_b	Bottom Adherend Thickness
t_t	Top Adherend Thickness
τ	Shear Stress
σ	Peel Stress
γ	Shear Strain
γ_e	Elastic Shear Strain
γ_p	Plastic Shear Strain
ϵ	Strain
ν	Poisson's Ratio
x	Longitudinal Coordinate with Origin in the Middle of the Overlap
X	Dimensionless Coordinate with Origin in the Middle of the Overlap

Chapter 1

Introdution

1.1 Background and motivation

Adhesives have been used for joining materials throughout history, but only recently has serious interest in this type of joining technique started to emerge as a true and valid replacement to other joining techniques such as bolting, riveting, welding, and others. To evaluate their true capabilities and find out if they can effectively be used to joint two different parts, there is the need to define their mechanical properties and behaviour as a joining material. Two ways exist to evaluate these parameters: using numerical methods or analytical methods.

In regard to analytical methods, there are many available, and their complexity ranges from a simple equation to systems of differential equations and iterative methods that require vast amounts of computational calculations to complete. The complexity of a method is therefore directly related to the time it takes to implement it in a computer program/spreadsheet, and until now there is not any pre-made software implementation of various analytical methods that allows to quickly evaluate an adhesive connection.

1.2 Objectives

The main objective of this thesis is to minimise the effort needed to correctly predict adhesive joint behaviour, by way of creating a simple to use interface where several analytical methods can be accessed. This way, an engineer does not need to take any time implementing all the equations of each method in their computer program of choice, but should only provide the joint characteristics (geometry, material properties, etc.) and choose an analytical method to obtain the desired results.

Specifically, the objectives are:

- Develop a web based interface that will serve as input (to define the joint's characteristics) and output (for displaying the results);
- Give the user the ability to use results in other places, by way of developing ways to export the computed data;
- Take advantage of the fact we are building in the web and implement extra features: sharing results with coworkers, save results online on the user private account so they can be accessed later, etc.
- Implement as many analytical methods as possible, covering the broadest possible range of situations: plastic and elastic analysis, obtaining stresses and failure loads, perform analysis on different types of joints.

1.3 Research methodology

To accomplish the objectives defined above, a methodology had to be defined. That methodology is the following

1. Do a bibliographic research on the topic of adhesive joints and ways to predict joint behaviour;
2. Investigate previous softwares and determine features that are already available;
3. Define a preliminar web interface that would make it possible to accomplish all the set objectives;
4. Define a modular structure for the analytical methods that works behind the web interface, making it easy to add new methods;
5. Propose at least one analytical method for each of the more important situations (obtain stresses for elastic analysis, obtain stresses for plastic analysis, obtain failure loads for elastic and plastic analysis, etc.);
6. Choose an external CAS to assist in the heavy calculations that more complex modules require;
7. Develop the web interface completely, connecting the analytical methods modules with the interface and programming the necessary extra functions: export results, print results, save analysis online, etc.

8. Think of ways to enrich the user experience and implement them: ability to change joint geometry values directly on the results screen and update results automatically, develop a database of adhesives and adherends so the user does not take time to enter all material properties, etc.

1.4 Thesis outline

The outline of this thesis is the following:

Chapter 2: An introductory literature review of adhesives in general is made, focusing on their historical development, some important definitions, advantages, disadvantages, analysis methods and a summary of existing applications whose purpose is to aid in the design of adhesive joints;

Chapter 3: All the analytical methods implemented in JointDesigner are described, and a comparison between the results of each method for a predefined adhesive joint is presented;

Chapter 4: A FEM method implementation is then discussed and explained;

Chapter 5: The web application development procedure is given, followed by the presentation of the final interface of JointDesigner;

Chapter 6 and 7: Conclusions and ideas for future work are presented.

Chapter 2

Literature review

2.1 Adhesive joints

2.1.1 History of adhesive bonding

Interest in adhesive joints has been rising in recent years due to the benefits that their usage can bring to mechanical systems, although adhesives have been around for thousands of years, with reports of their usage dating all the way back to 4000BC [1]. The timeline present in Figure 2.1 shows some of the occurrences related to adhesive use throughout history.

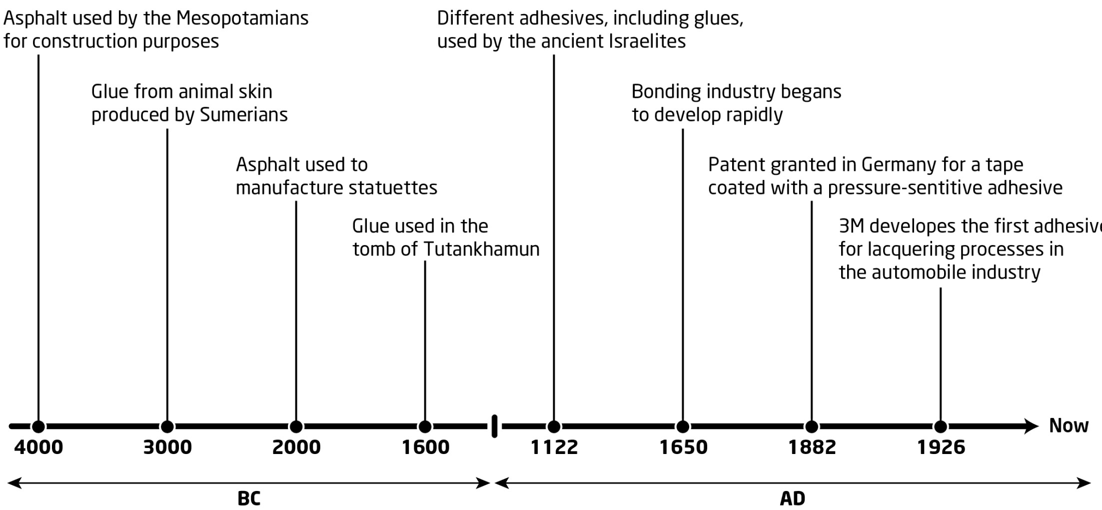

Figure 2.1: Adhesives throughout history [1].

In the United States, adhesives started to be patented and manufactured in the early nineteenth century, and Table 2.1 shows some of the most important occurrences.

Table 2.1: Chronological developments of adhesives in the United States [2].

Year	Material
1814	Glue from animal bones (patent)
1872	Domestic manufacture of fish glues (isinglass)
1874	First U.S. fish glue patent
1875	Laminating of thin wood veneers attains commercial importance
1917	Casein glues for aircraft construction
1927	Cyclized rubber in adhesives (Fischer-Goodrich Co.)
1930	Specialty pressure-sensitive tapes: rubber base (Drew–Minnesota Mining & Mfg. Co.)
1944	Metal-bond adhesives (Havens, Consolidated Vultee-Aircraft Corp.)

The twentieth century marked the beginning of commercially available adhesives, as Table 2.2 shows.

Table 2.2: Commercial availability of adhesives in the twentieth century [3].

Year	Adhesive
1910	Phenol-formaldehyde
1930	Urea-formaldehyde
1940	Nitrile-phenolic, vinyl-phenolic, acrylic, polyurethane
1950	Epoxies, cyanoacrylates, anaerobics
1960	Polyimide, polybenzimidazole, polyquinoxaline
1970	Second-generation acrylic

Due to their susceptibility to temperature, in the middle of the twentieth century adhesives have started to be improved in this regard, and Figure 2.2 clearly shows an increasing trend-line related to the maximum service temperature of adhesives:

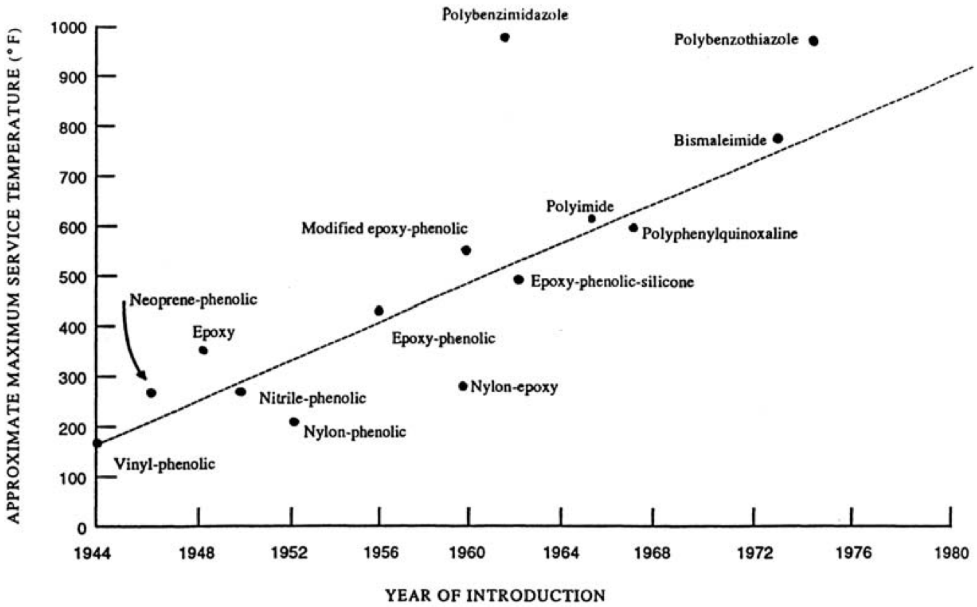

Figure 2.2: Maximum service temperature improvements for adhesives over the recent years [2].

2.1.2 Definitions

Adhesion is not an easy word to characterise, and an entirely satisfactory definition is yet to be found [4]. The following definition has been proposed [5]:

> "*Adhesion* refers to the state in which two dissimilar bodies are held together by intimate interfacial contact such that mechanical force or work can be transferred across the interface. The interfacial forces holding the two phases together may arise from van der Waals forces, chemical bonding, or electrostatic attraction. Mechanical strength of the system is determined not only by the interfacial forces, but also by the mechanical properties of the interfacial zone and the two bulk phases."

After an adhesive joint has been created, it can be solicited in 4 different ways:

- Compression (Figure 2.3)
- Tension (Figure 2.4)
- Shear (Figure 2.5)
- Peel (Figure 2.6)

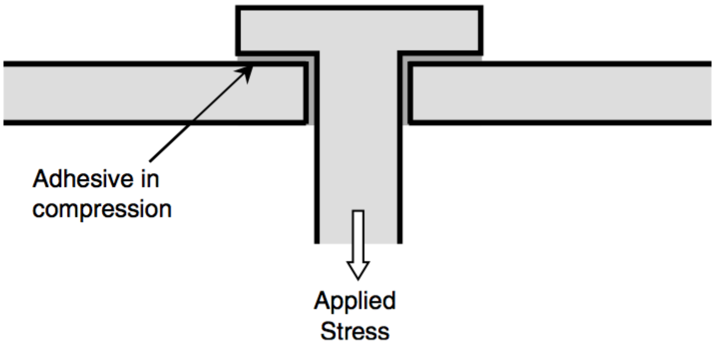

Figure 2.3: An example of a joint design which encourages compressive loads rather than tensile forces [6].

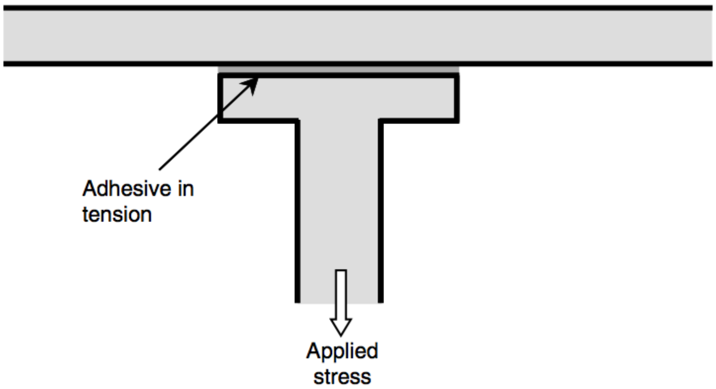

Figure 2.4: The tensile load is more destructive in terms of adhesive bonding [6].

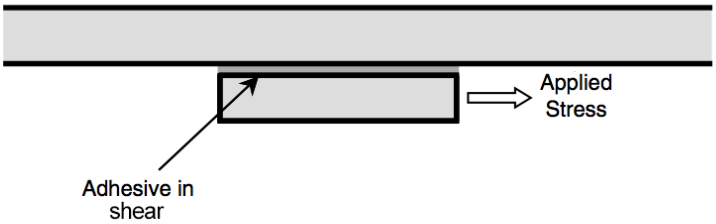

Figure 2.5: Shear conditions applied to an adhesive connection.

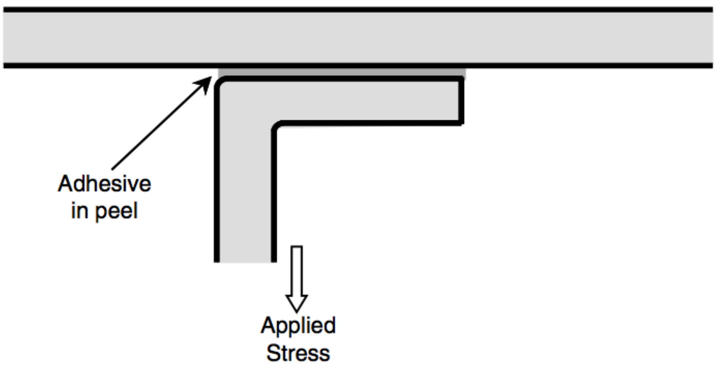

Figure 2.6: Under peeling conditions adhesive performance will be much reduced [6].

There are two principle types of adhesive bonding [4]:

Structural adhesion: specific for applications where the adherends can experience large stresses, they are defined as adhesives having a shear strength greater than 7 MPa and must also have significant resistance to ageing;

Non-structural adhesion: used merely to hold lightweight materials and are not required to support substantial loads.

2.1.3 Advantages

Structural adhesive bonding has the following main advantages [7, 8]:

- Stress concentrations are substantially inferior to those present in bolted or riveted connections, as load is transferred through the whole of the bonded area, whereas bolting and riveting create discrete load points through which load must be transferred and therefore increasing the stress concentration in those points (see Figure 2.7);

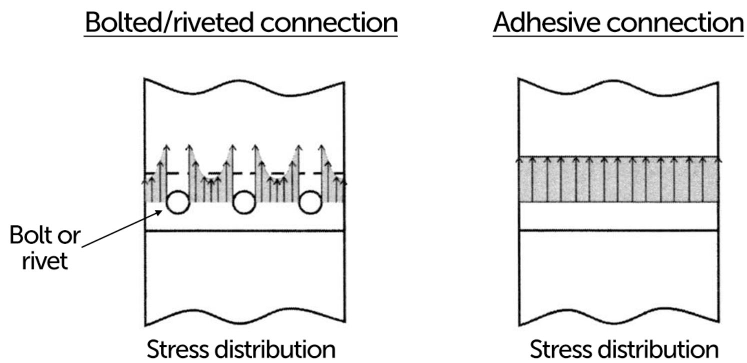

Figure 2.7: Comparison between the stress distribution in a riveted connection and an adhesive connection [7].

- The low density of adhesives, due to their polymeric nature, makes them more appealing for connecting components in weight efficient structures. Examples of industries where this concern is key are the aerospace and automotive industry;

- A connection can be achieved for different materials, which are not necessarily metallic. Composite components can therefore be joined and adhesives are the only viable option for those types of connections (bolting and riveting would interfere with the fibres of composites, and welding cannot be used due to its metal deposition nature);

- Variable thickness bond-lines can relax or eliminate manufacturing tolerance requirements in the components' connection area, reducing manufacturing times and increasing productivity;

- Vibration damping is another important advantage of adhesive joints, as they can much more easily elastically deform and regain the initial configuration than other joining methods (see Figure 2.8).

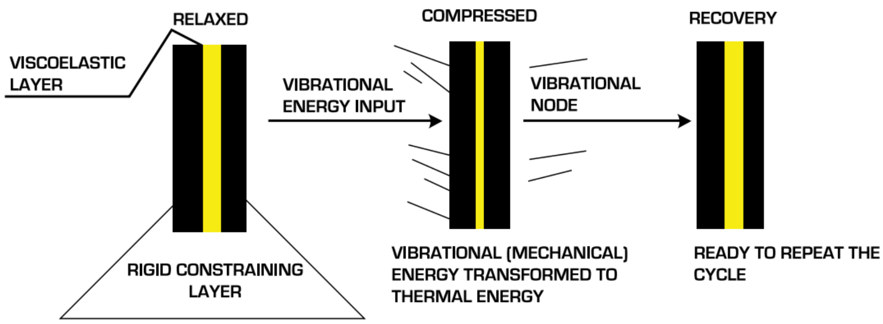

Figure 2.8: Response of a viscoelastic adhesive to vibrational input [9].

2.1.4 Disadvantages

Naturally, every technology presents disadvantages, and adhesive connections are no exception. Although work is being constantly done to improve this bonding technique and lessen the negative properties of adhesive bonding, the following main disadvantages exist [7, 8]:

- Temperature is an important aspect that must be taken into consideration when selecting and designing an adhesive connection. Due to their polymeric nature, adhesives are much more susceptible to temperature, whereas bolts and rivets made of metal have very different (and relaxed) temperature requirements. Figure 2.9 shows that vulnerability to temperature;

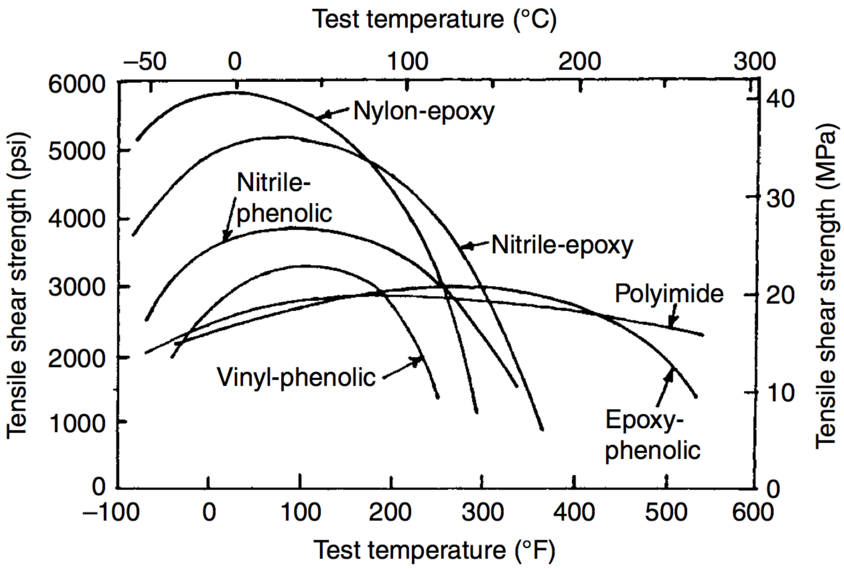

Figure 2.9: Tensile shear stress as a function of temperature for various types of adhesives [4].

- The type of solicitation that an adhesive is subject to must be carefully studied, because different types of failure stresses exist depending on the direction of solicitation. As can be seen in Figure 2.10, the peel failure stress is the one with lowest value, and therefore loads that subject the adhesive joint to peel stresses must be carefully analysed. Figure 2.11 shows some situations where the adhesive joint has poor resistance and how it can be improved (so that the adhesive is not subjected to peel stresses);

- Humidity is another important aspect as adhesives absorb moisture through their life, which can cause weakening and result in premature failure;

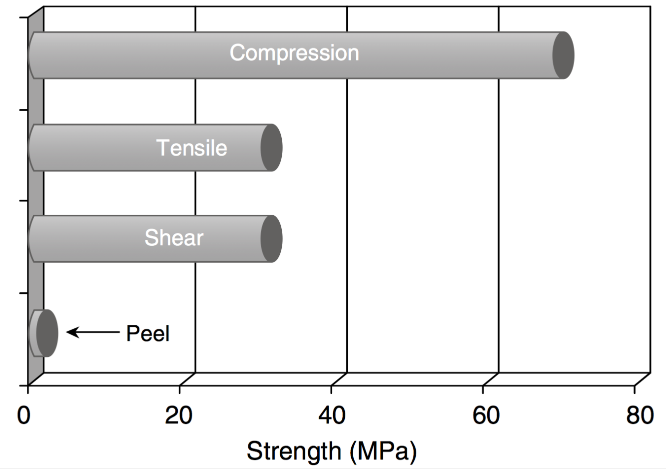

Figure 2.10: Comparison of failure stresses for a high-performance adhesive [6].

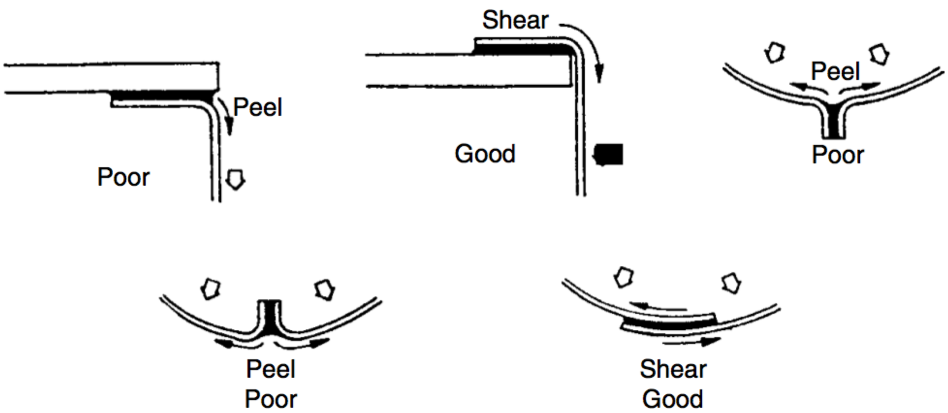

Figure 2.11: Improving an adhesive joint resistance (substituting peel solicitations to shear solicitations)[4].

- Adhesives are very limited in respect to maintenance considerations of the mechanical components. When these need to be separated, adhesives must be destroyed to disconnect the components, leading to increased maintenance costs;
- Surface roughness is an important consideration because there may exist gas pockets or voids in the adhesive connection, especially if the surface energy of the adherend is low (Figure 2.12), resulting in a crack propagation from one pocket to the next and causing the joint to break;
- It is hard to detect defects related to poor adhesion in adhesive bonds without the use of destructive methods;
- The prediction of an adhesive joint behaviour is complex and depends on various factors, which leads to different analytical methods whose complexity can require a simple calculator or a powerful computer for solving.

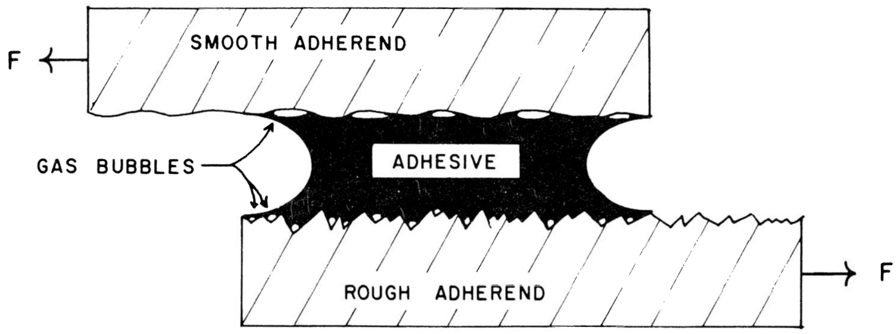

Figure 2.12: Surface roughness effect on the existence of gas pockets [10].

The present thesis, as already stated, aims to minimise this last disadvantage by way of creating a user interface to handle all heavy calculations and present the necessary results to the prediction of the joint behaviour.

2.2 Analysis methods

To calculate the stresses and other data pertaining to specific adhesive joints, there are two main methods used.

2.2.1 Analytical methods

Analytical methods allow that fast calculations are obtained for simple and well known geometries. These analyses are based on the solution of differential equations formulated for a specific joint configuration [3]. Each analytical method has its advantages and disadvantages, and is specific to some material behaviour and geometry. If we use the correct analytical method to the problem at hand, results obtained have enough precision and can be used with a certain level of accuracy to predict the adhesive joint behaviour.

There are several analytical methods that can handle:

- Material linearity and nonlinearity for both adhesive and adherends;
- Isotropic or composite adherends, and different material and geometrical properties for each adherend;

This means that there are several analytical methods that can be used in a variety of situations [11, 12, 13, 14], and they should be used if the problem being studied fits their requirements. Not unlike the FEM, some of the more advanced analytical methods require considerable amounts of computations to reach a solution, and should be used only when necessary.

2.2.2 Finite Element Method

The FEM is a numerical method to determine the approximate solution to boundary value problems. By way of discretizing the domain at study, in a mesh of various finite elements, it is then possible to numerically find the solutions for each element, and therefore globally for the system being studied. The main advantage of the FEM is that structures of high complexity can be studied, as the problem is discretized in small elements where stresses are easily obtained, and the disadvantage is the amount of time it takes to calculate the results for all the elements. In our case, where we study a mechanical system, we are able to find displacements and stresses, but the FEM analysis can be used in other problems like thermal analysis, fluid flow analysis, piezoelectric analysis, and many others [15].

Using FEM for the study and analysis of adhesive joints is useful in various scenarios, some of those being [3]:

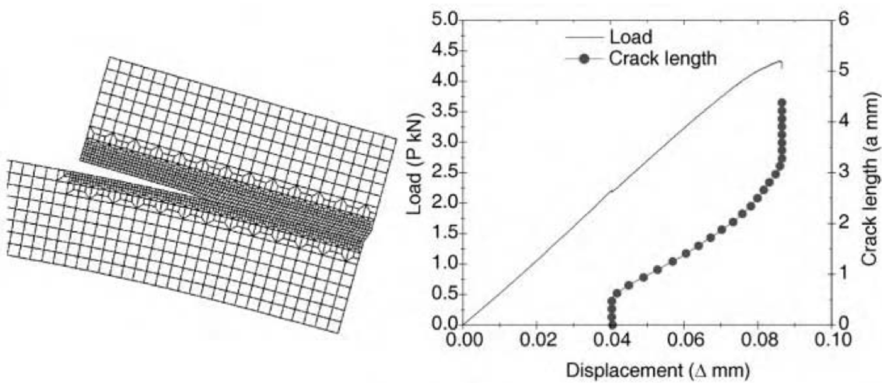

Figure 2.13: Stable crack growth predicted for a degraded single lap joint using the FEM [3].

- Complex geometric configurations;
- Complex material responses;
- Coupled multi-physics problems;
- Local bi-material stress singularities;
- High-speed dynamic events.

The disadvantages are mainly [16]:

- The time taken to calculate the whole problem is directly dependent on the number of elements that exist in the mesh. Because of that, the more we refine the mesh in certain areas, the longer it takes for the calculation to end, but the results are more accurate. A compromise between speed and accuracy must therefore be taken into consideration;
- The results obtained are never exact, they are always an approximation. Most of the time those approximations are more than enough, but the engineer must be aware of these small details as to correctly design an adhesive joint.

Because of all this, FEM should be used when there are to analytical methods that can solve our problem, or when we wish a higher level of accuracy at the expense of computational time.

2.3 Software for the design of adhesive joints

Commercially there is only one software solution [17] that provides implementations of some analytical methods and provides results pertaining to the behaviour of an adhesive joint. There are several other applications that address specific situations (Table 2.3), but are not easy to use.

2.3.1 JointDesigner

One of the applications previously developed was a visual interface named *JointDesigner*, in which this thesis bases itself upon. *JointDesigner* was written in MATLAB, and presented the following interface to the user:

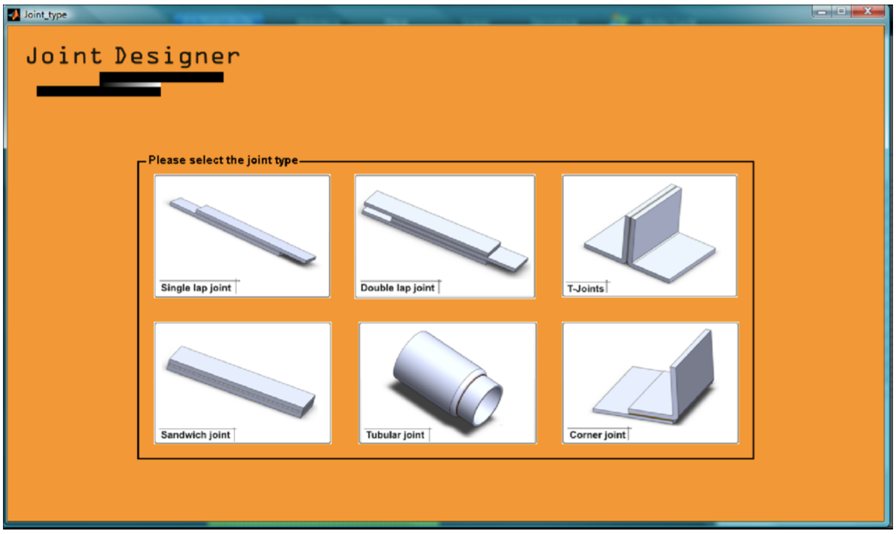

Figure 2.14: JointDesigner interface

The aforementioned software implemented various features that were available for the user. The joint types and analytical models implemented were the following:

Joint Types:

Single lap, double lap and sandwich joints;

Models:

Volkersen [18], Goland & Reissner [19], Hart-Smith [20], Bigwood & Crocombe [21], and Adams et al [22].

Other features pictured in Figure 2.15 included the ability to define how many data points the user would like to plot, a function to export the numerical data to other applications (by way of exporting to an text file), and the ability to print resulting charts.

This application will be further mentioned in the next chapter.

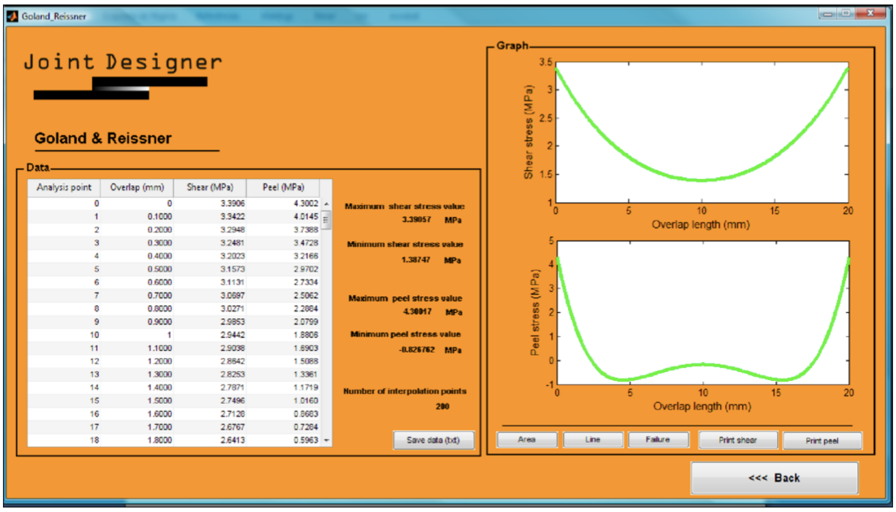

Figure 2.15: Example of JointDesigner results for the Goland & Reissner model

2.3.2 Other software

Some other applications exist that address specific situations in the design of adhesive joints. These are displayed in Table 2.3.

Table 2.3: Software Packages Available in the Market [23].

Name (Supplier)	Application	Features
BOLT (G.S. Springer, Stanford University)	Design of pin-loaded holes in composites	• Prediction of failure strength and failure mode • Three types of bolted joints: Joints with a single hole, Joints with two identical holes in a row, Joints with two identical holes in tandem • Applicable to uniform tensile loads and symmetric laminates
BISEPSLOCO (AEA Technology,UK.)	Closed form computer code for predicting stresses and strains in adhesively bonded single-lap joints	• Tensile/shear/bending moment loading • Adhesive peel and shear stress predictions • Allowance for plasticity in adhesive layer • Thermal stress analysis
BISEPSTUG (AEA Technology, UK)	Closed form computer code for predicting stresses and strains in adhesively bonded coaxial joints	• Stepped and profiled joints • Orthotropic adherends • Torsional and axial loading • Allowance for plasticity in adhesive layer • Thermal stress analysis
CoDA (National Physical Laboratory, UK)	Preliminary design of composite beams and panels, and bolted joints	• Synthesis of composite material properties (lamina and laminates for a range of fibre formats) • Parametric analyses • Panel and beam design • Bonded and bolted double shear joints • Bearing, shear-out, pin shear and by-pass tensile failure prediction

Name (Supplier)	Application	Features
DLR (DLRMitteilung, Germany)	Preliminary design of composite joints	• Adhesively bonded and bolted joints • Linear-elastic and linear-elastic/plastic behaviour • Tension and shear loading • Symmetric and asymmetric lap joints • Bearing, shear-out, pin shear and by-pass tensile failure prediction. (washers and bolt tightening)
FELOCO (AEA Technology, UK)	Finite element module computer code for predicting stresses and strains in adhesively bonded lap shear joints	• Stepped and profiled joints • Tensile/shear/bending moment/pressure loading • Linear and non-linear analysis • Peel, shear and longitudinal stress predictions in adhesive layer and adherends
PAL (Permabond, UK)	"Expert" system for adhesive selection	• Thermal stress analysis for adherend and adhesive joined systems include: Lap joints, butt joints, sandwich structures, bushes, gears, bearings, shafts, pipe, threaded fittings • Elastic analysis • Creep/fatigue effects on joint stiffness (graphical)
RETCALC (Loctite, UK)	Interactive windows based software general purpose	• Joint strength • Correction factors (temperature and fatigue)Stepped and profiled joints

Chapter 3

Analytical methods implemented

As was said in the section regarding the disadvantages of adhesives, this thesis aims to provide an easy to use interface where analytical methods for various situations can be employed. Those analytical methods are presented next.

3.1 Volkersen

The first analytical method presented is the simplest, as it computes only the shear stress and ignores some details that occur in an adhesive connection. In 1938, Volkersen [18] introduced the concept of differential shear (Figure 3.1), in which the adherend was modeled as a rod undergoing axial deformation only and the adhesive as a continuous shear spring.

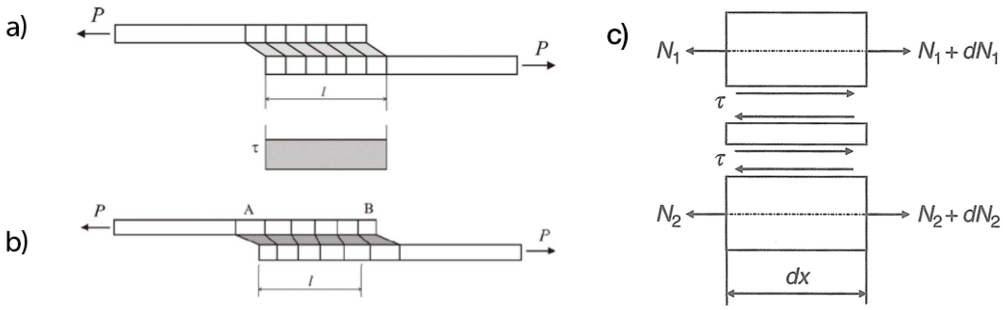

Figure 3.1: Comparison of deformations in load SLJs: a) rigid adherends; b) elastic adherends; c) free body diagrams of the Volkersen's model [24].

Because of these assumptions the moment that naturally occurs on single lap joints due to the non-collinearity of the applied forces is discarded, which makes this method more representative of DLJs behaviour than SLJs, because the bending of the adherends is not as significant in DLJs. The reduction of the strain in the adherends along the overlap and the continuity of the adhesive/adherend interface cause a non-uniform shear strain (and stress) distribution in the adhesive layer. The shear stress is maximum at the ends of the overlap and is much lower at the middle (Figure 3.2).

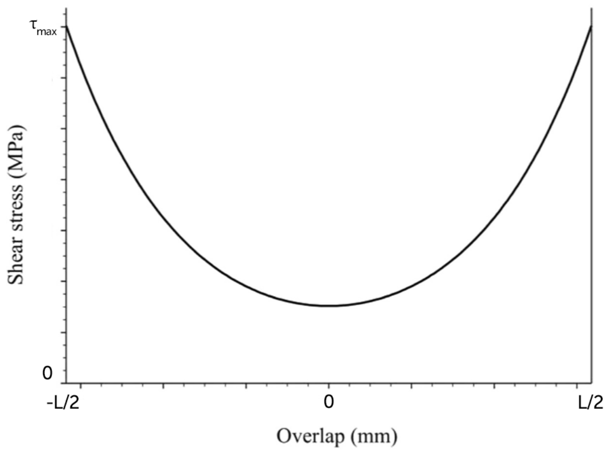

Figure 3.2: Example of a shear stress distribution obtained through the Volkersen method.

The shear stress distribution is given by the following equation:

$$\tau(X) = \frac{Pw}{2bl}\frac{\cosh(wX)}{\sinh(w/2)} + (\frac{\psi - 1}{\psi + 1})\frac{w}{2}\frac{\sinh(wX)}{\cosh(w/2)} \tag{3.1}$$

Where:

$$w^2 = (1+\psi)\phi$$

$$\psi = t_t/t_b$$

$$\phi = \frac{G_a l^2}{E t_t t_a}$$

$$X = x/l \qquad \text{where} \quad -\frac{1}{2} \leq X \leq \frac{1}{2}$$

3.2 Goland & Reissner

In 1944, Goland & Reissner [19] proposed an adhesive-beam model to analyse the SLJ, in which it used Euler beams to model the adherends, and a two-parameter elastic medium to model the adhesive. It considered the bending moment and transverse force that occurs due to the non-collinearity of the applied load (Figure 3.3a) and d)), which causes the joint to rotate until the applied loads tend to align (Figure 3.3b)). The rotation occurs as to relieve the bending moment, M, at the joint edges. Because of this effect, the bending moment decreases as the joint rotates further, giving rise to a nonlinear geometric problem where the effects of the large deflections of the adherends must be accounted for.

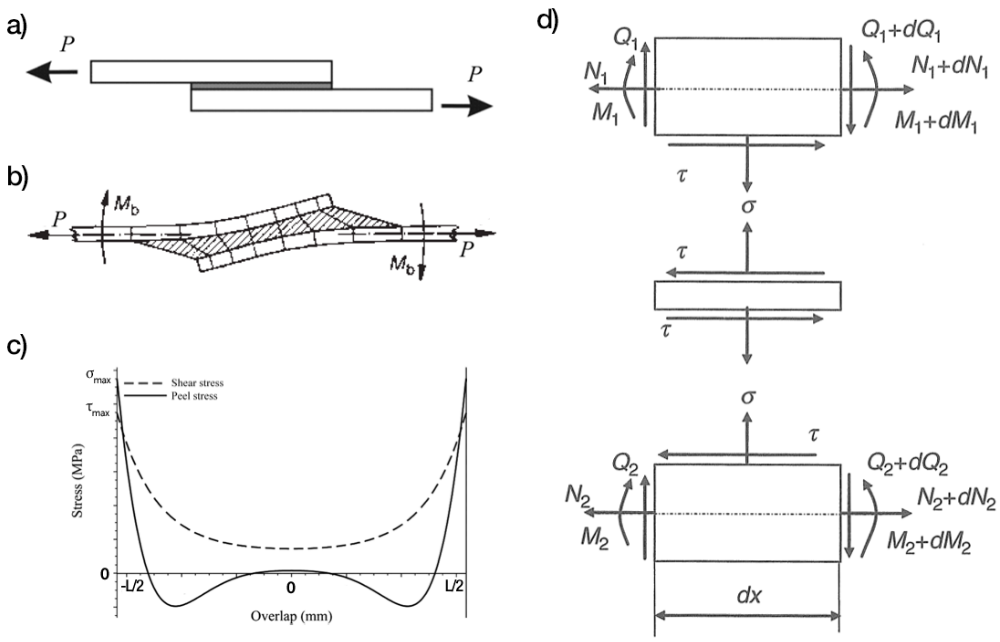

Figure 3.3: Goland & Reissner's model: a) undeformed joint; b) deformed joint with thin adherends; c) resulting plots of the analysis (shear and peel stress); d) free body diagrams of a linear overlap.

The Goland & Reissner's method uses a bending moment factor (k) and a transverse force factor (k') that relates the applied tensile load per unit width (P) to the bending moment (M) and the transverse force (V) at the overlap ends If the joint does not rotate, i.e. for very small applied loads, the factors k and k' are approximately equal to 1. As the joint rotates with the increase of load, k and k' decreases and, consequently, the bending moment and the transverse load decreases too. Goland & Reissner took into account the effect of large deflections of the adherends, but assumed that the adherends were integral, with an infinitely thin adhesive layer.

The adhesive shear stress is given by:

$$\tau(x) = -\frac{\overline{P}}{8c}\left\{\frac{\beta c}{t}(1+3k)\frac{\cosh\left(\frac{\beta x}{t}\right)}{\sinh\left(\frac{\beta c}{t}\right)} + 3(1-k)\right\} \tag{3.2}$$

Where:

$$\beta^2 = 8\frac{G_a}{E}\frac{t}{t_a}$$
$$k = \frac{\cosh(u_2 c)}{\cosh(u_2 c) + 2\sqrt{2}\sinh(u_2 c)}$$
$$u_2 = \sqrt{\frac{3(1-\nu^2)}{2}}\frac{1}{t}\sqrt{\frac{\overline{P}}{tE}}$$

The adhesive peel stress is given by:

$$\begin{aligned}\sigma(x) = \frac{\overline{P}t}{\Delta c^2}\Bigg[&\left(R_2\lambda^2\frac{k}{2} + \lambda k'\cosh(\lambda)\cos(\lambda)\right)\cosh\left(\frac{\lambda x}{c}\right)\cos\left(\frac{\lambda x}{c}\right)\\ &+\left(R_1\lambda^2\frac{k}{2} + \lambda k'\sinh(\lambda)\sin(\lambda)\right)\sinh\left(\frac{\lambda x}{c}\right)\sin\left(\frac{\lambda x}{c}\right)\Bigg]\end{aligned} \tag{3.3}$$

Where:

$$\lambda = \gamma c/t$$
$$\gamma^4 = 6\frac{E_a}{E}\frac{t}{t_a}$$
$$k' = \frac{kc}{t}\sqrt{3(1-\nu^2)\frac{\overline{P}}{tE}}$$
$$R_1 = \cosh(\lambda)\sin(\lambda) + \sinh(\lambda)\cos(\lambda)$$
$$R_2 = \sinh(\lambda)\cos(\lambda) - \cosh(\lambda)\sin(\lambda)$$
$$\Delta = \frac{1}{2}(\sin(2\lambda) + \sinh(2\lambda))$$

The solution for the shear and normal stresses in the adhesive layer is found through solving differential equations which satisfy compatibility and equilibrium throughout the joint. To facilitate a solution, shear and normal deformations in the adherends were ignored. In spite of its deficiencies, Goland & Reissner's work served to illustrate the importance of the bending moment and therefore the need to consider normal (or peel) stresses [25].

3.3 Ojalvo & Eidinoff

Ojalvo & Eidinoff [26] detected a deficiency in the work published by Goland & Reissner [19] 34 years before. The problem was an incomplete shear-strain/displacement equation for the adhesive, which, because of the pioneer work done by Goland & Reissner, perpetuated errors ever since. Because of that, Ojalvo & Eidinoff corrected the mistake in a way that does not complicate the analysis to any significant degree, and also studied the effect of the correction.

The shear stress equation is:

$$\tau(x) = A\cosh(\lambda\sqrt{2+6(1+\beta)^2}x) + B \tag{3.4}$$

where the constants A and B are obtained from differentiation and integration of (3.4) and substitution into:

$$\begin{aligned} \frac{d}{dx}\tau &= 2\lambda^2\left[1+3(1+\beta)^2k\right] \\ \int \tau dx &= 2 \end{aligned}$$

The peel stress equation is:

$$\sigma(x) = C\sinh(\alpha_1 x)\sin(\alpha_2 x) + D\cosh(\alpha_1 x)\cos(\alpha_2 x) \tag{3.5}$$

where the constants C and D are obtained upon substitution of the derivatives of (3.5) into:

$$\begin{aligned} \frac{d}{dx^3}\sigma - 6\beta\lambda^2\frac{d}{dx}\sigma &= k\gamma\rho^2(1+\beta) \\ \frac{d}{dx^2}\sigma &= k\gamma\rho^2(1+\beta) \end{aligned}$$

The various variables are defined as follows:

$$\begin{aligned} \alpha_1^2 &= \frac{3\beta\lambda^2+\rho}{2} \\ \alpha_2^2 &= \frac{-3\beta\lambda^2+\rho}{2} \\ \gamma &= t/2c \\ \beta &= h/t \\ \rho^2 &= \frac{24E_a c^4}{Eht^3} \\ \lambda^2 &= \frac{G_a c^2}{Eht} \end{aligned}$$

$$k = \frac{M_0}{(N_0 t/2)(1+\beta)}$$

Where M_0 and N_0 are as Figure 3.4 shows.

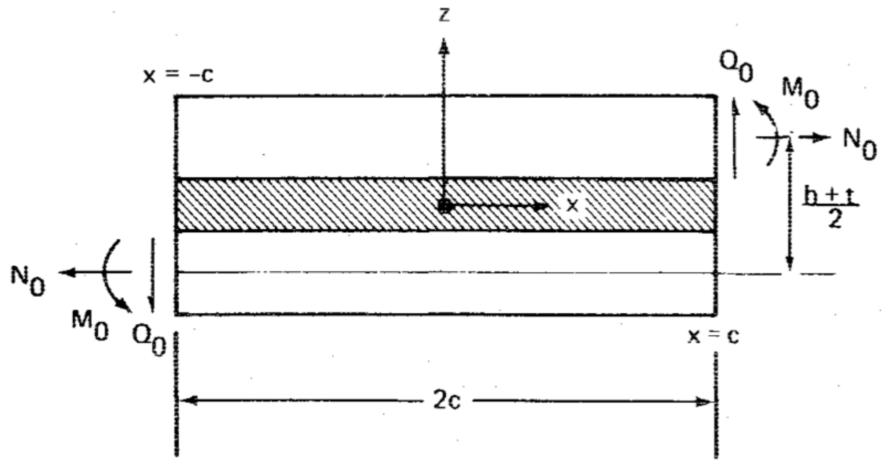

Figure 3.4: Internal adherend loads at the ends of the joint region [26].

A comparison of their results with Goland & Reissner's yields the shear and peel plots visible in Figure 3.5.

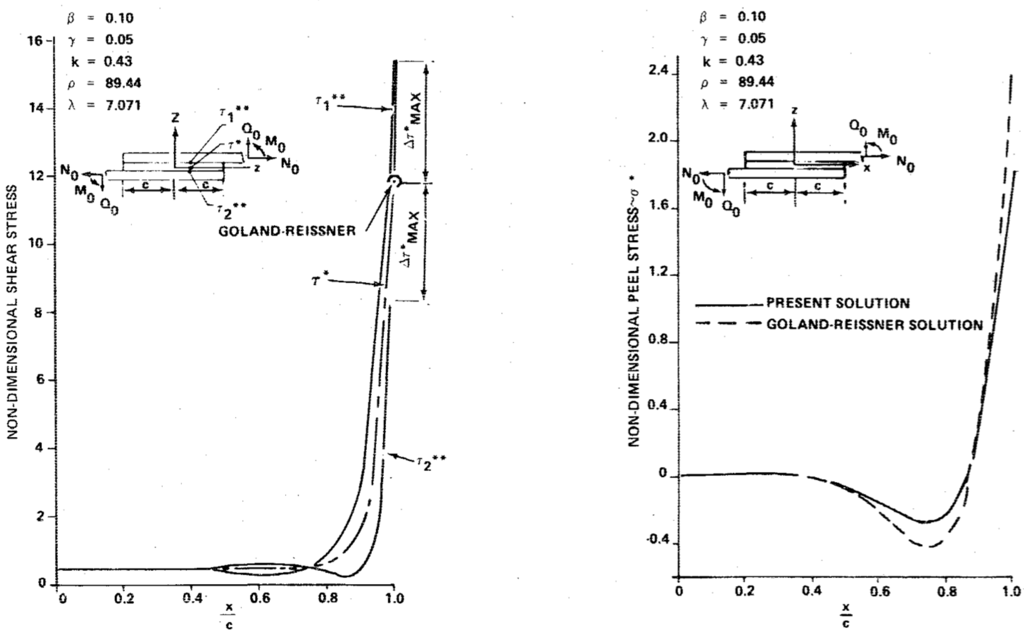

Figure 3.5: Comparison between Ojalvo & Eidinoff and Goland & Reissner's results: (left) shear stress distribution, and (right) peel stress distribution [26].

3.4 Hart Smith

In 1973, Hart–Smith [20] took the study of joint analysis further by considering the plastic deformation of an adhesive in addition to the elastic response. This work, coupled with the finite element analysis technique provided the platform for calculation of stress distribution in complex nonlinear joints.

If we allow for adhesive plasticity, the joint strength prediction is higher than for an elastic analysis because a larger failure strain is used. The maximum lap joint strength was calculated by using the maximum shear strain as the failure criterion. Any differences between the adherends result in a decrease of the joint strength.

To characterise the adhesive behaviour, Hart-Smith chose an elasto-plastic model (Figure 3.6) such that the ultimate shear stress and strain in the model are equal to the ultimate shear stress and strain of the real stress-strain curve of the adhesive, the two curves having the same strain energy. The maximum lap joint strength was calculated by using the maximum shear strain as the failure criterion, as shown in Figure 3.6.

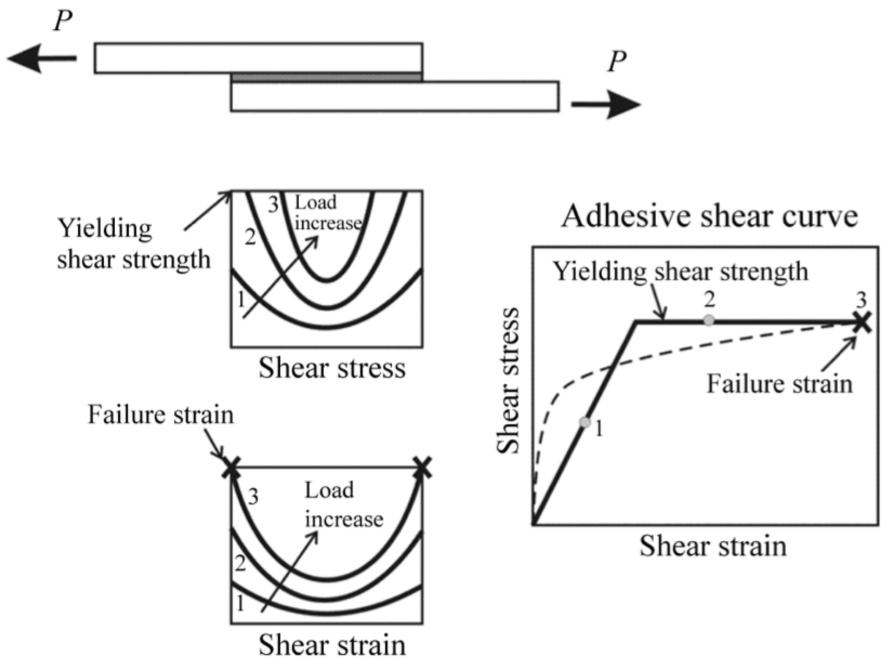

Figure 3.6: Plasticity in the adhesive according to Hart-Smith [7].

Hart-Smith [20] gives a closed-form algebraic solution for the elastic shear and peel adhesive stresses. The origin of x is the middle of the overlap. The adhesive shear stress is given by:

$$\tau(x) = A_2 \cosh(2\lambda' x) + C_2 \tag{3.6}$$

Where:

$$\begin{aligned}
\lambda' &= \sqrt{\left[\frac{1+3(1-\nu^2)}{4}\right]\frac{2G_a}{t_a E t}} \\
A_2 &= \frac{G_a}{t_a E t}\left[\overline{P} + \frac{6(1-\nu^2)M}{t}\right]\frac{1}{2\lambda' \sinh(2\lambda' c)} \\
C_2 &= \frac{1}{2c}\left[\overline{P} - \frac{A_2}{\lambda'}\sinh(2\lambda' c)\right] \\
M &= \overline{P}\left(\frac{t+t_a}{2}\right)\frac{1}{1+\xi c+(\xi^2 x^2/6)} \\
\xi^2 &= \overline{P}/D \\
D &= Et^3/12(1-\nu^2)
\end{aligned}$$

The adhesive peel stress is given by:

$$\sigma(x) = A\cosh(\chi x)\cos(\chi x) + B\sinh(\chi x)\sin(\chi x) \tag{3.7}$$

Where:

$$\begin{aligned}
\chi^4 &= E_a/2Dt_a \\
A &= -\frac{E_a M[\sin(\chi c) - \cos(\chi c)]}{t_a D\chi^2 e^{(\chi c)}} \\
B &= \frac{E_a M[\sin(\chi c) + \cos(\chi c)]}{t_a D\chi^2 e^{(\chi c)}}
\end{aligned}$$

Hart-Smith also considered adhesive shear stress plasticity, keeping the peel stress elastic. The shear stress is modelled using a bi-linear elastic-perfectly plastic approximation. The overlap is divided into three regions, a central elastic region of length d and two outer plastic regions. The overlap length is l and, for a balanced lap-joint, both non- linear regions have length $(l-d)/2$. Coordinates x and x' are defined in these regions, as shown in Figure 3.7.

The problem is solved in the elastic region in terms of the shear stress according to:

$$\tau(x) = A_2 \cosh(2\lambda' x) + \tau_p(1-K) \tag{3.8}$$

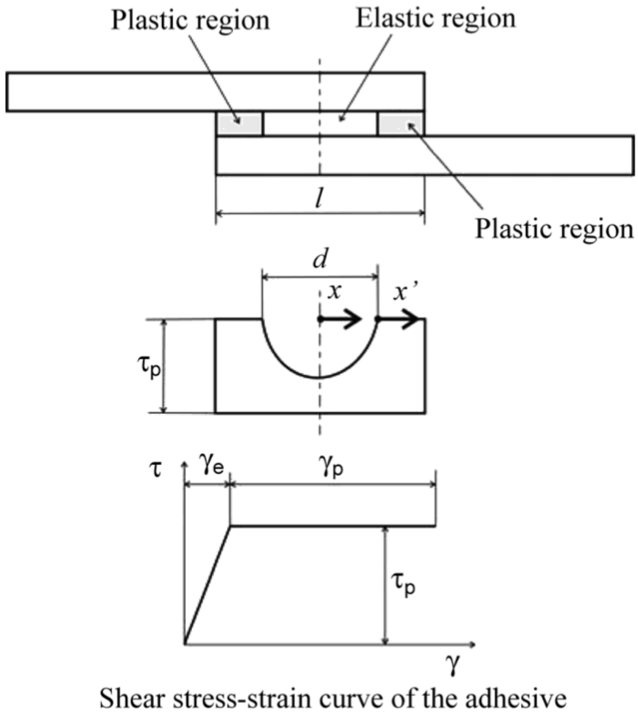

Figure 3.7: Plasticity in the adhesive according to Hart-Smith [23].

and the shear strain in the plastic region according to:

$$\gamma(x') = \gamma_e\{1 + 2K[(\lambda' x')^2 + \lambda' x' \tanh(\lambda' d)]\} \tag{3.9}$$

where τ_p is the plastic adhesive shear stress and

$$A_2 = \frac{K\tau_p}{\cosh(\lambda' d)} \tag{3.10}$$

K and d are solved iteratively using the following equations:

$$\frac{P}{l\tau_p}(\lambda' l) = 2\lambda'\left(\frac{l-d}{2}\right) + (1-K)(\lambda' d) + K\tanh(\lambda' d) \tag{3.11}$$

$$\frac{P}{\tau_p}\lambda^2\left[1 + 3k(1-\nu^2)\left(1 + \frac{t_a}{t}\right)\right] = 2\left(\frac{\gamma_p}{\gamma_e}\right) + K\left[2\lambda'\left(\frac{l-d}{2}\right)\right]^2 \tag{3.12}$$

$$2\left(\frac{\gamma_p}{\gamma_e}\right) = K\left\{\left[2\lambda'\left(\frac{l-d}{2}\right) + \tanh(\lambda' d)\right]^2 - \tanh(\lambda' d)\right\} \tag{3.13}$$

where γ_e is the elastic adhesive shear strain and γ_p the plastic adhesive shear strain (see Figure 3.7). An initial value of the bending moment factor, k, is given and the system solved for P, K, and d. This process is repeated until there is convergence of k.

3.5 Bigwood & Crocombe

Bigwood & Crocombe published two important types of analysis: linear analysis [21], and non-linear analysis [27]. Only the linear analysis is available in JointDesigner due to some complications in the implementation of the non-linear analysis.

In an attempt to rationalise the analysis of bonded joints and enable a general analysis to be derived which can consider various configurations of adhesive joint under complex loading, the overlap region was reduced to a simple adherend-adhesive sandwich, as shown in Figure 3.8. Using this sandwich it is possible to analyse any joint that can be simplified to this form, and for which the prescribed end loading parameters can be calculated. Some of the suitable configurations are presented in Figure 3.9.

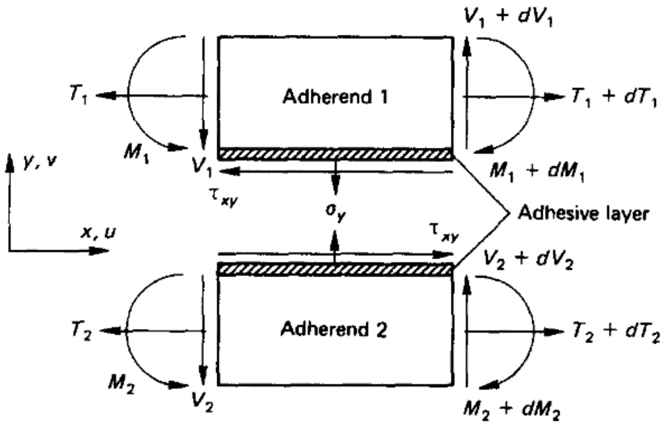

Figure 3.8: Elemental diagram of an adherend-adhesive sandwich under general loading (loading values per unit width) [21].

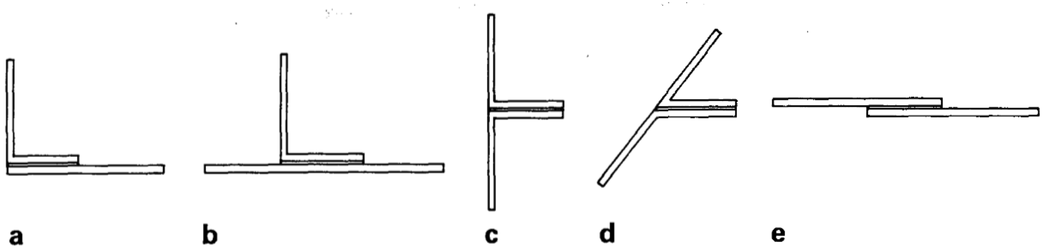

Figure 3.9: Joint configurations suitable for analysis: (a) L-joint; (b) T-joint; (c) T-peel joint; (d) Inclined T-peel joint; and (e) single lap joint [21].

Simplified peel and shear equations were proposed, which are, respectively:

$$\begin{aligned}\sigma(x) &= A_1 \cos(K_5 x) \cosh(K_5 x) + A_2 \cos(K_5 x) \sinh(K_5 x) \\ &\quad + A_3 \sin(K_5 x) \cosh(K_5 x) + A_4 \sin(K_5 x) \sinh(K_5 x) \\ \tau(x) &= B_1 \cosh(K_6 x) + B_2 \sinh(K_6 x) + B_3\end{aligned} \tag{3.14}$$

All constants (A_{1-4}, B_{1-3} and K_{5-6}) are given in the paper [21] and are fully defined.

3.6 Adams

This analytical method is different from the ones presented until now, because instead of stresses it allows the determination of the failure load for both adhesive and adherends. Adams et al. [22, 28, 29] proposed a simple predictive model that gives the failure load for adhesive global yielding and adherend yielding. For substrates that yield, a plateau is reached for a certain value of overlap corresponding to the yielding of the adherend, the joint strength being easily predicted. For intermediate or brittle adhesives and non- yielding adherends, the analysis is less robust and the author suggests using the finite element method or a more complete analytical solution. The failure load of the adhesive joint (P_a), with elastic adherends, corresponds to the total plastic deformation of the adhesive (i.e., everywhere in yield), and the maximum load that can be carried which just creates adherend yield corresponds to the failure load of the adhesive joint (P_s).

The design methodology is represented graphically in Figure 3.10.

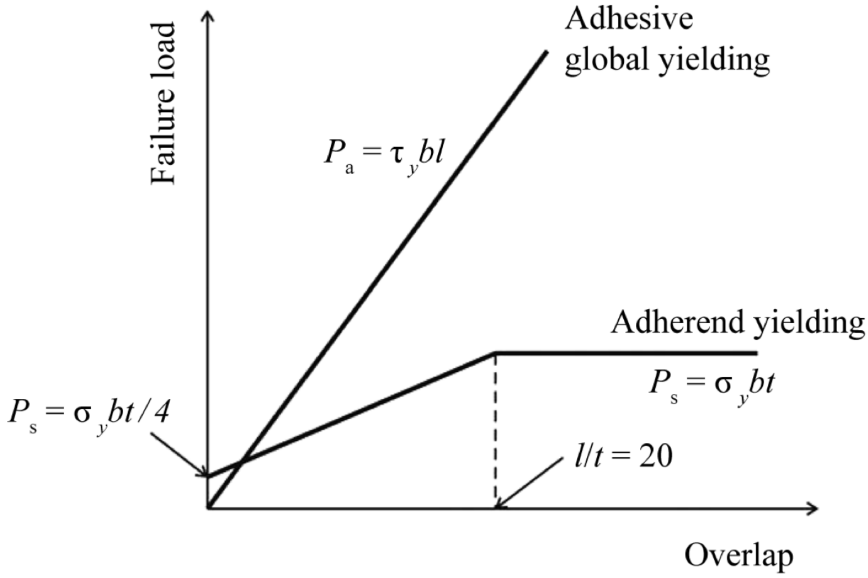

Figure 3.10: Simple design methodology of single lap joints based on the adherend yielding method according to Adams et al. [7].

In a SLJ with elastic adherends, the load corresponding to the total plastic deformation of the adhesive (i.e., every- where in yield) is:

$$P_a = \tau_y bl \tag{3.15}$$

Where P_a is the failure load of the adhesive joint and τ_y is the yield strength of the adhesive.

The direct tensile stress (σ_t) acting in the adherend due to the applied load, P, is:

$$\sigma_t = P/bt \tag{3.16}$$

If there is bending (as per Goland and Reissner [19]), the stress at the inner adherend surface (σ_s) due to the bending moment, M, is:

$$\sigma_s = 6M/bt^2 \tag{3.17}$$

Where $M = kPt/2$ [19]. The variable k is the bending moment factor which reduces (from unity) as the lap rotates under load. The stress acting in the adherend is the sum of the direct stress and the bending stress. Thus, the maximum load which can be carried which just creates adherend yield (P_s) is:

$$P_s = \sigma_y bt/(1 + 3k) \tag{3.18}$$

Where σ_y is the yield strength of the adherend. For low loads and short overlaps, k is approximately 1. Therefore, for such a case:

$$P_s = \sigma_y bt/4 \tag{3.19}$$

However, for joints which are long compared with the adherend thick- ness, such that $l/t \geq 20$, the value of k decreases and tends to zero. In this case, the whole of the cross-section yields in tension and P_s is:

$$P_s = \sigma_y bt \tag{3.20}$$

3.7 Summary

The following table summarises the analytical methods implemented and their capabilities:

Table 3.1: Analytical methods and the status of their implementation in the web application.

Model	Elastic Adhesive	Plastic Adhesive	Elastic Adherends	Plastic Adherends	Isotropic Adherends	Results Given	Implemented
Volkersen	x		x		x	Shear Stress	Yes
Goland & Reissner	x		x		x	Shear and Peel Stresses	Yes
Ojalvo & Eidinoff	x		x		x	Shear and Peel Stresses	Yes
Hart Smith	x	x	x		x	Adhesive Shear, Adhesive Peel, Plastic Shear Stresses and Plastic Shear Strain	Yes
Bigwood & Crocombe Linear	x		x		x	Shear and Peel Stresses	Yes
Adams	x	x	x	x	x	Failure Load	Yes

3.8 Comparison between models

The methods that were previously mentioned as implemented in the web application naturally present differences between each other (otherwise there would be no reason for them to exist), so it seems appropriate that a comparison is made between results for each one of them for the same joint geometry .

For that purpose, a joint with the following characteristics was analysed:

Table 3.2: Properties of the joint used for the comparison between analytical methods.

Variable	Property name	Value (unit)
l	Overlap Length	12.7 (mm)
b	Joint Width	25.4 (mm)
t	Adherends Thickness	1.62 (mm)
t_a	Adhesive Thickness	0.25 (mm)
E	Adherends Young's Modulus	70 (GPa)
ν	Adherends Poisson's Ratio	0.3
E_a	Adhesive Young's Modulus	4.82 (GPa)
ν_a	Adhesive Poisson's Ratio	0.4
F	Load Intensity	1000 (N)

The following image of a SLJ illustrates the places where we compared results between methods:

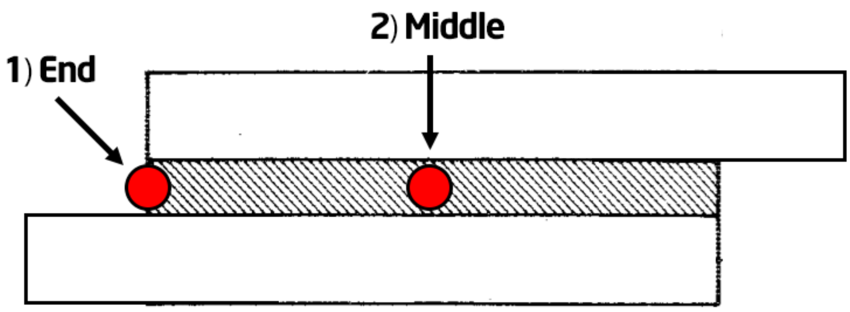

Figure 3.11: Locations where comparison of the results for the different methods were made.

3.8.1 End of the overlap

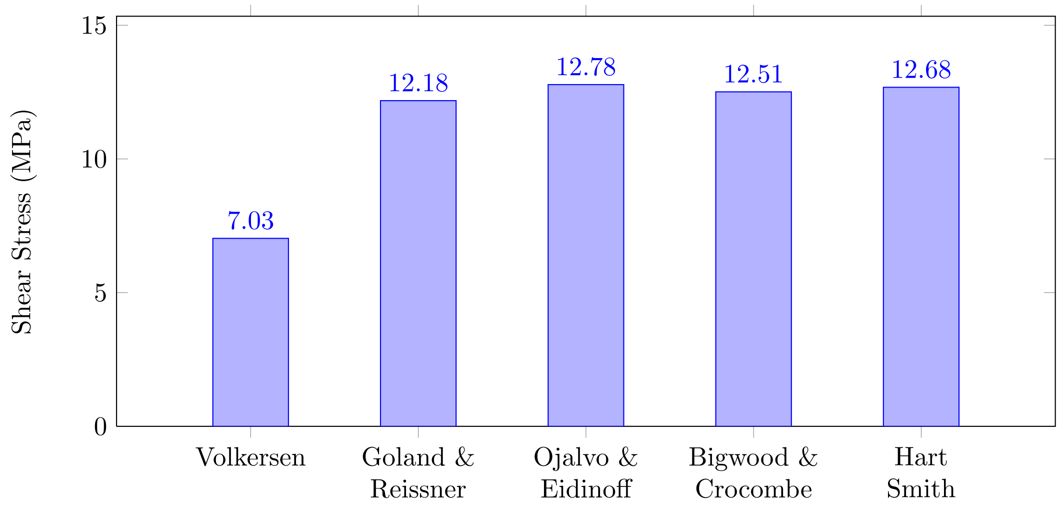

Figure 3.12: Comparison of shear stresses at the end of the overlap.

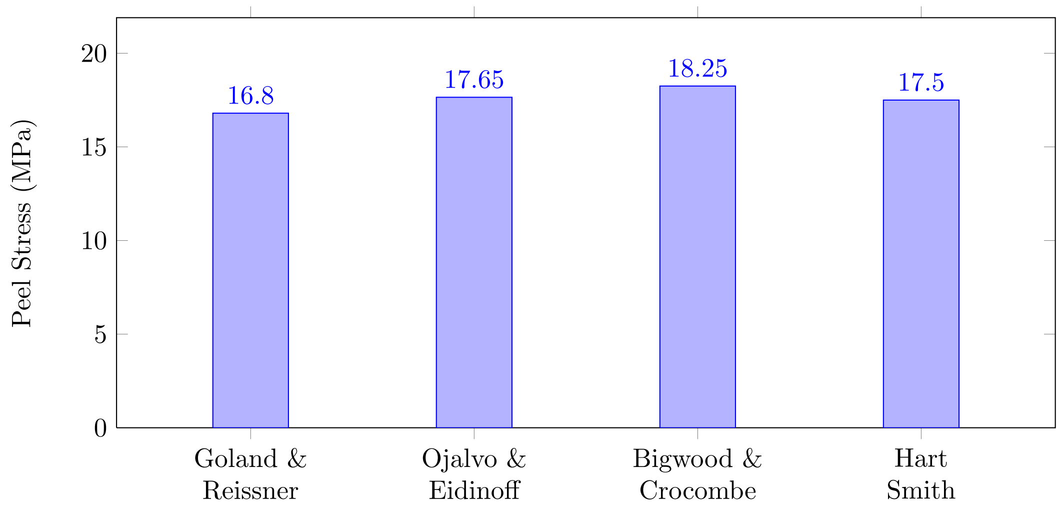

Figure 3.13: Comparison of peel stresses at the end of the overlap.

3.8.2 Middle of the overlap

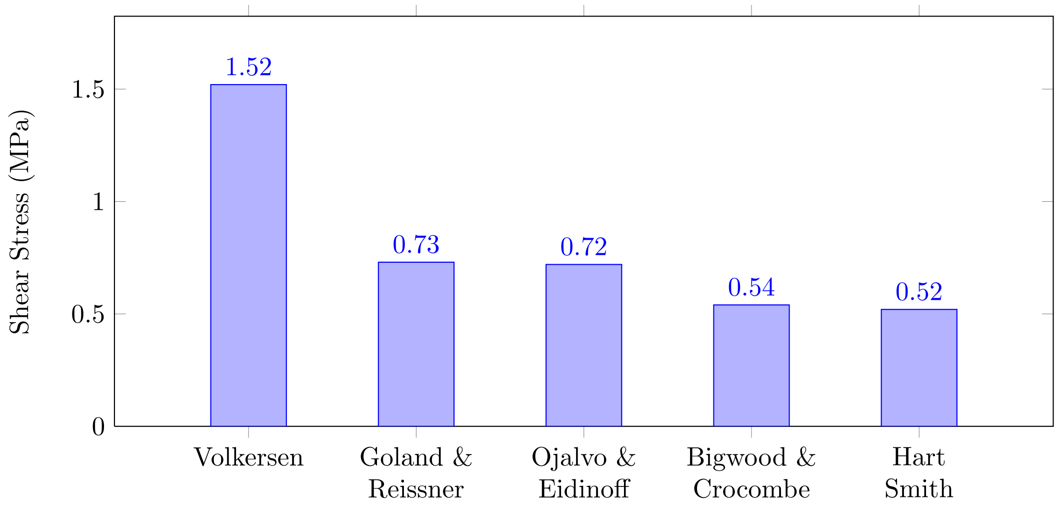

Figure 3.14: Comparison of shear stresses at the middle of the overlap.

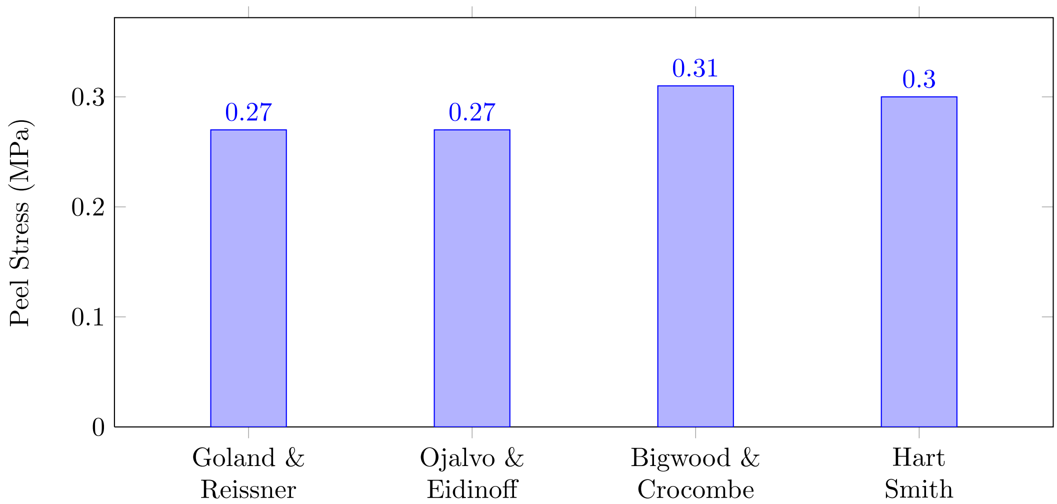

Figure 3.15: Comparison of peel stresses at the middle of the overlap.

3.8.3 Discussion

Regarding the shear stress, it is clear that the Volkersen's method results in values considerably different from the remaining analytical methods. The reason for this is that it assumes several simplifications, as mentioned previously, and when we discard both the peel stresses and the bending moment that naturally occurs in SLJs subjected to tension (bending also increases the shear stress), naturally the results will be farther from reality than if we considered those parameters. Nonetheless, the Volkersen method provides a first approximation to the shear stresses we can expect, and its simple formulation makes it a safe initial approximation.

Still with regard to the shear stress plots, it is clear that all methods besides Volkersen present very similar values. This is because their formulations are closer to reality, and therefore all the similar results basically validate each other.

In the peel stresses domain, Volkersen is naturally excluded from the comparison, and all the values are very similar as well, which again confirms their validity. It should be noted that the peel stresses at the middle of an overlap are expected to be zero or very close to it, which is also proved by the results.

With the previously shown plots it is clear that the analytical methods are correctly implemented in JointDesigner.

Chapter 4

FEM model implementation

4.1 Reasons for a FEM model

As previously said, the FEM is very useful to study complex geometries and problems that are not solvable by the existing closed-form models, but at the expense of time. In fact, the total time it takes to obtain results through the FEM is the cumulative sum of the time it takes for each of the following taks to complete:

- Define the type of finite element to use;
- Model the problem, specifically the geometry and material properties;
- Define a mesh and refine it at the appropriate places;
- Define the boundary conditions and applied loads;
- Run the solver and obtain results.

Because of all these steps, the engineer needs to take some time to correctly set up everything and double check the data. As the purpose of this application is to facilitate the joint design process, it was clear that a custom FEM implementation is beneficial, where all common tasks are handled automatically, and thus a custom FEM was created to be included with JointDesigner. The key advantages are:

- Automatically choose the finite element type for the problem (2D or 3D);
- Model the problem automatically by use of inputed information by the user (joint type, dimensions and material properties);

- Define an appropriate mesh and refine it in places where singularities occur (for example, at the edges of the overlap a singularity exists, and so the node distribution is higher);
- Boundary condition and loads to the joint are automatically applied to the correct nodes.

Then, a solver is used to obtain results and display them to the user.

4.2 Implementation

The implementation section describes all the steps that were taken to produce all the necessary code for the implementation of the custom FEM method. It shows the formulation of a 4 node rectangular finite element, which is used to elastically analyse a SLJ. There was no time to implement further things in the method, like a cohesive element, plastic analysis or temperature considerations.

4.2.1 Theory

The FEM analysis consists in using a specific method to solve the discretized system at hand. There are various methods that can be used to reach a FEM solution, but the most common is the Direct Stiffness Method (DSM) [30]. In the DSM, we find the solution to each Degree Of Freedom (DOF) in the system by way of solving the following system of equations:

$$[K]\{\delta\} = \{P\} \tag{4.1}$$

Where $[K]$ represents the global stiffness matrix of the system, $\{\delta\}$ the vector of unknowns that are the value of each DOF, and $\{P\}$ the external forces applied to the system. The steps required to solve a system through the FEM are:

1. Define the problem dimensions and how many degrees of freedom will be studied;
2. Distribute nodes through the problem domain in a way that areas of special interest have more nodes than areas without interest;
3. Connect the nodes through elements appropriate to the problem being studied, e.g. rectangular element of 4 nodes;
4. Compute the local stiffness matrix of each element in matrix form;
5. Assemble the local stiffness matrix of each element in the global stiffness matrix of the problem $[K]$;

6. Determine which nodes are affected by the applied forces and compute vector $\{P\}$;

7. Solve the system of equations (4.1) for $\{\delta\}$.

Figure 4.1 illustrates the previous discretization steps applied to a single lap joint analysis:

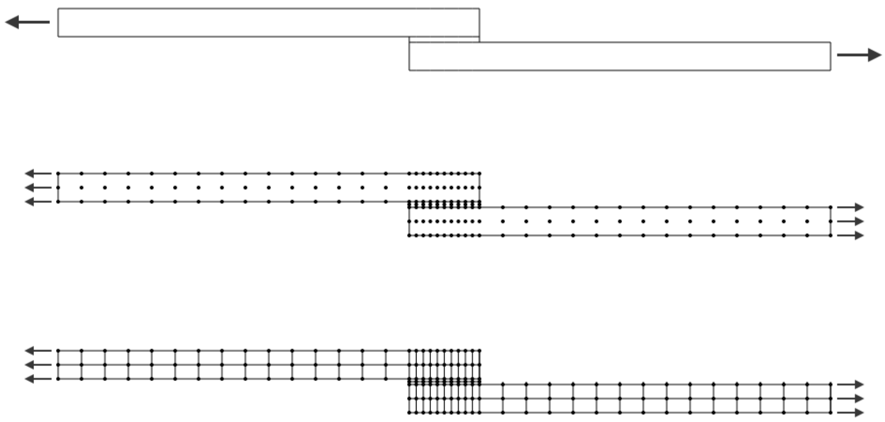

Figure 4.1: Discretization of a single lap joint problem: top) SLJ to be analysed, middle) nodes are inserted through the SLJ joint and the applied load is distributed to the appropriate nodes, bottom) finite elements created by connecting them to the nodes.

4.2.2 Problem geometry

Firstly, the dimensions of the SLJ parameters are defined: the adherents width, length and thickness, and the adhesive thickness as well as the overlap width. With these parameters we can:

- Determine the correct node distribution (i.e. the spacing between nodes so they are all equally spaced), which is naturally dependent on the joint geometry;
- Obtain a visual representation of the joint, as shown in Figure 4.1.

4.2.3 Nodes

The node distribution parameters were defined as the following:

- Nodes are inserted left to right, top to bottom;
- Node placement depends on several parameters:
  - Number of nodes horizontally outside the adhesive joint connection;
  - Number of nodes horizontally in the adhesive joint connection (preferably a higher count, as this is the area of interest);

– Number of nodes vertically both outside and in the adhesive joint connection.

Thanks to all these separate parameters the node count can be independently set and fine tuned, and the application can then proceed to calculate the total number of nodes and their position on the geometry, by distributing the node count on the defined dimensions. The result of this process is visualised in the following figure:

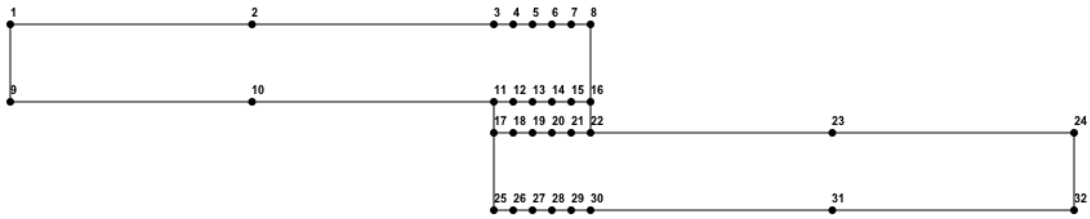

Figure 4.2: Example of a node distribution and count in a SLJ.

4.2.4 Elements

After creating the nodes, the next step is to connect them through finite elements. Only 2D analysis is available in the first version, and thus 4 node elements were chosen with 2 degrees of freedom in each node.

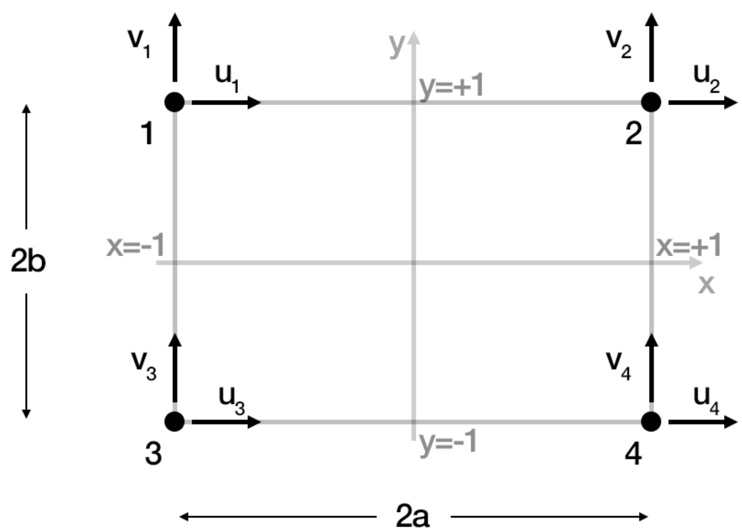

Figure 4.3: 4 node element used, each node with 2 degrees of freedom (8 DOF per element). Dimensions of the element are $2a$ horizontally, and $2b$ vertically. Axis xOy are represented in light gray, whose values are between -1 and $+1$.

It can be seen in Figure 4.3 that there are 2 degrees of freedom per node: DOF u_i represents the displacement of node i in the x direction, and v_i represents the displacement of node i in the y direction. The coordinates of each node are displayed in Table 4.1.

Table 4.1: Coordinates of each of the nodes for the finite element represented in Figure 4.3.

Node (i)	x_i	y_i
1	-1	$+1$
2	$+1$	$+1$
3	-1	-1
4	$+1$	-1

In future versions where plastic analysis and crack control will be available, other element types will be needed. As for now, only an elastic analysis is considered, and thus the aforementioned element type is used for all finite elements.

4.2.5 Matrices definition

FEM bases itself upon the mathematical calculation of matrices and vectors, whose values relate to physical properties of the materials. First, we define the shape functions that correlate the displacements of the finite element with the global displacement of the lap joint. We have 4 nodes per element (Figure 4.3), thus we need 4 shape functions:

$$\begin{aligned} N_1 &= \frac{1}{4}\,(1-x)\,(1+y) \\ N_2 &= \frac{1}{4}\,(1+x)\,(1+y) \\ N_3 &= \frac{1}{4}\,(1-x)\,(1-y) \\ N_4 &= \frac{1}{4}\,(1+x)\,(1-y) \end{aligned} \tag{4.2}$$

Shape functions are created so that for the node they represent, for example N_1 represents node 1, the value they take is 1, and for all other nodes the value is 0. For example:

$$\begin{aligned} N_1(x=-1, y=+1) &= \frac{1}{4}(1-(-1))(1+(+1)) = \frac{1}{4}(2)(2) = 1 \\ N_1(x=+1, y=+1) &= \frac{1}{4}(1-(+1))(1+(+1)) = \frac{1}{4}(0)(2) = 0 \\ N_1(x=-1, y=-1) &= \frac{1}{4}(1-(-1))(1+(-1)) = \frac{1}{4}(2)(0) = 0 \\ N_1(x=+1, y=-1) &= \frac{1}{4}(1-(+1))(1+(-1)) = \frac{1}{4}(0)(0) = 0 \end{aligned} \tag{4.3}$$

Similar checks can be made for all remaining shape functions. We can now build matrix $[N]$, the *matrix of the shape functions*, using the information from (4.2), thus:

$$[N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \tag{4.4}$$

And matrix $[B]$, which represents the *strain matrix*, is the following:

$$[B] = [L][N] = \begin{bmatrix} d/dx & 0 \\ 0 & d/dy \\ d/dy & d/dx \end{bmatrix} \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \tag{4.5}$$

Where $[L]$ represents the *differential operation matrix.* With this matrix defined, we can calculate the element stiffness matrix, $[k_e]$, whose dimension is $[n \times n]$, where n is the number of DOFs of the element:

$$[k_e] = \int_A h[B]^T[c][B]dA = \int_{-1}^{1}\int_{-1}^{1} h[B]^T[c][B]dxdy \tag{4.6}$$

In Equation (4.6) A represents the area of the element (Figure 4.3), e the element, h the element thickness (on the plane perpendicular to xOy), $[B]^T$ the transposed matrix of $[B]$, and $[c]$ the *matrix of material constants*, which needs to be defined. For isotropic materials we have:

$$\begin{aligned} [c] &= \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \text{(Plane stress)} \\ [c] &= \frac{E(1-\nu)}{(1+\nu)(1-2\nu}\begin{bmatrix} 1 & \nu/(1-\nu) & 0 \\ \nu/(1-\nu) & 1 & 0 \\ 0 & 0 & (1-2\nu)/2(1-\nu) \end{bmatrix} \text{(Plane strain)} \end{aligned} \tag{4.7}$$

We now have everything we need to calculate the element stiffness matrix, $[k_e]$ (4.6).

4.2.6 Computing the element stiffness matrix

It would be a matter of computing the integrals to determine the stiffness matrix. In reality, doing two integral operations per element for a system with thousands of elements would take a long time, and to handle that problem a process called *Gauss Integration* is employed.

Gauss Integration is a method to obtain an approximate value of an integral without actually computing it, but instead evaluating the expression to integrate at specific key points and assigning a weight to each of those key points. Mathematically, Gauss Integration is defined as:

$$g = \int_{-1}^{1} f(x)dx \approx \sum_{j=1}^{n} w_j f(x_j) \tag{4.8}$$

For different values of n, the following table summarises the key points and respective weights:

Table 4.2: Gauss integration points and weight coefficients [15][30].

n	x_i	w_i
1	0	2
2	$-1/\sqrt{3}$; $1/\sqrt{3}$	1 ; 1
3	$-\sqrt{0.6}$; 0 ; $\sqrt{0.6}$	5/9 ; 8/9 ; 5/9

Visually this translates to the following:

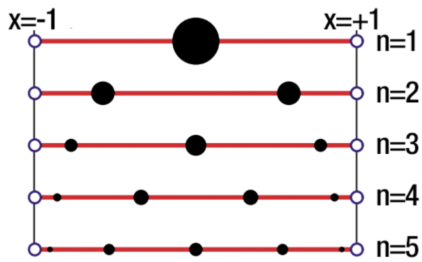

Figure 4.4: The first five one-dimensional Gauss rules n = 1,2,3,4,5 depicted over the line segment $x \in [-1, +1]$. Sample point locations are marked with black circles. The radius of those circles are proportional to the integration weights [30].

So for example, if we wish to compute an integral with $n = 2$, it would be:

$$g = \int_{-1}^{1} f(x)dx \approx f(x_1)w_1 + f(x_2)w_2 = f(-1/\sqrt{3}) \times 1 + f(1/\sqrt{3}) \times 1 \tag{4.9}$$

With this procedure we obtain very good approximations (the higher the n, theoretically the better approximation) and save time on expensive integral computations. In reality, $n = 2$ provides a very acceptable approximation for our purposes.

4.2.7 Assembly of the global stiffness matrix

Now that we have each individual element's stiffness matrix, $[k_e]$, we need to assemble them in the global stiffness matrix $[K]$, so we can then solve (4.1).

Let's consider the following example:

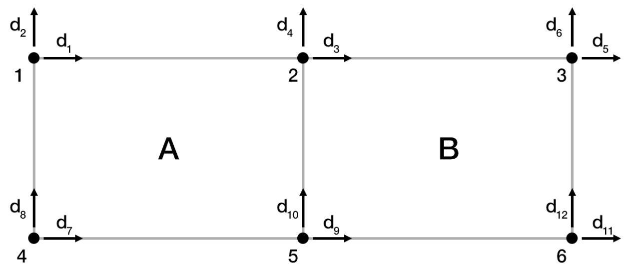

Figure 4.5: Representation of a problem discretized by two elements, A and B, with 6 total nodes and 12 global DOFs.

Each element has a stiffness matrix of $[8\times 8]$ non-zero elements, because the element connects to 8 DOFs. In the case of element A, it connects to nodes 1, 2, 4 and 5, and therefore it relates to DOFs 1, 2, 3, 4, 7, 8, 9 and 10.

$$[k_A] = \begin{bmatrix} K_{A_{1,1}} & K_{A_{1,2}} & K_{A_{1,3}} & K_{A_{1,4}} & K_{A_{1,7}} & K_{A_{1,8}} & K_{A_{1,9}} & K_{A_{1,10}} \\ K_{A_{2,1}} & K_{A_{2,2}} & K_{A_{2,3}} & K_{A_{2,4}} & K_{A_{2,7}} & K_{A_{2,8}} & K_{A_{2,9}} & K_{A_{2,10}} \\ K_{A_{3,1}} & K_{A_{3,2}} & K_{A_{3,3}} & K_{A_{3,4}} & K_{A_{3,7}} & K_{A_{3,8}} & K_{A_{3,9}} & K_{A_{3,10}} \\ K_{A_{4,1}} & K_{A_{4,2}} & K_{A_{4,3}} & K_{A_{4,4}} & K_{A_{4,7}} & K_{A_{4,8}} & K_{A_{4,9}} & K_{A_{4,10}} \\ K_{A_{7,1}} & K_{A_{7,2}} & K_{A_{7,3}} & K_{A_{7,4}} & K_{A_{7,7}} & K_{A_{7,8}} & K_{A_{7,9}} & K_{A_{7,10}} \\ K_{A_{8,1}} & K_{A_{8,2}} & K_{A_{8,3}} & K_{A_{8,4}} & K_{A_{8,7}} & K_{A_{8,8}} & K_{A_{8,9}} & K_{A_{8,10}} \\ K_{A_{9,1}} & K_{A_{9,2}} & K_{A_{9,3}} & K_{A_{9,4}} & K_{A_{9,7}} & K_{A_{9,8}} & K_{A_{9,9}} & K_{A_{9,10}} \\ K_{A_{10,1}} & K_{A_{10,2}} & K_{A_{10,3}} & K_{A_{10,4}} & K_{A_{10,7}} & K_{A_{10,8}} & K_{A_{10,9}} & K_{A_{10,10}} \end{bmatrix} \quad (4.10)$$

With this, we can expand matrix (4.10) to a dimension of $[12\times 12]$, the total number of DOFs of the problem, simply using the value 0 for DOFs than do not belong to element A:

$$
[k_A] = \begin{bmatrix}
K_{A_{1,1}} & K_{A_{1,2}} & K_{A_{1,3}} & K_{A_{1,4}} & 0 & 0 & K_{A_{1,7}} & K_{A_{1,8}} & K_{A_{1,9}} & K_{A_{1,10}} & 0 & 0 \\
K_{A_{2,1}} & K_{A_{2,2}} & K_{A_{2,3}} & K_{A_{2,4}} & 0 & 0 & K_{A_{2,7}} & K_{A_{2,8}} & K_{A_{2,9}} & K_{A_{2,10}} & 0 & 0 \\
K_{A_{3,1}} & K_{A_{3,2}} & K_{A_{3,3}} & K_{A_{3,4}} & 0 & 0 & K_{A_{3,7}} & K_{A_{3,8}} & K_{A_{3,9}} & K_{A_{3,10}} & 0 & 0 \\
K_{A_{4,1}} & K_{A_{4,2}} & K_{A_{4,3}} & K_{A_{4,4}} & 0 & 0 & K_{A_{4,7}} & K_{A_{4,8}} & K_{A_{4,9}} & K_{A_{4,10}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K_{A_{7,1}} & K_{A_{7,2}} & K_{A_{7,3}} & K_{A_{7,4}} & 0 & 0 & K_{A_{7,7}} & K_{A_{7,8}} & K_{A_{7,9}} & K_{A_{7,10}} & 0 & 0 \\
K_{A_{8,1}} & K_{A_{8,2}} & K_{A_{8,3}} & K_{A_{8,4}} & 0 & 0 & K_{A_{8,7}} & K_{A_{8,8}} & K_{A_{8,9}} & K_{A_{8,10}} & 0 & 0 \\
K_{A_{9,1}} & K_{A_{9,2}} & K_{A_{9,3}} & K_{A_{9,4}} & 0 & 0 & K_{A_{9,7}} & K_{A_{9,8}} & K_{A_{9,9}} & K_{A_{9,10}} & 0 & 0 \\
K_{A_{10,1}} & K_{A_{10,2}} & K_{A_{10,3}} & K_{A_{10,4}} & 0 & 0 & K_{A_{10,7}} & K_{A_{10,8}} & K_{A_{10,9}} & K_{A_{10,10}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{4.11}
$$

We now have the stiffness matrix of element A in the global form. If we do the same with element B, which connects to DOFs 3, 4, 5, 6, 9, 10, 11 and 12 we obtain:

$$
[k_B] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_{B_{3,3}} & K_{B_{3,4}} & K_{B_{3,5}} & K_{B_{3,6}} & 0 & 0 & K_{B_{3,9}} & K_{B_{3,10}} & K_{B_{3,11}} & K_{B_{3,11}} \\
0 & 0 & K_{B_{4,3}} & K_{B_{4,4}} & K_{B_{4,5}} & K_{B_{4,6}} & 0 & 0 & K_{B_{4,9}} & K_{B_{4,10}} & K_{B_{4,11}} & K_{B_{4,11}} \\
0 & 0 & K_{B_{5,3}} & K_{B_{5,4}} & K_{B_{5,5}} & K_{B_{5,6}} & 0 & 0 & K_{B_{5,9}} & K_{B_{5,10}} & K_{B_{5,11}} & K_{B_{5,11}} \\
0 & 0 & K_{B_{6,3}} & K_{B_{6,4}} & K_{B_{6,5}} & K_{B_{6,6}} & 0 & 0 & K_{B_{6,9}} & K_{B_{6,10}} & K_{B_{6,11}} & K_{B_{6,11}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_{B_{9,3}} & K_{B_{9,4}} & K_{B_{9,5}} & K_{B_{9,6}} & 0 & 0 & K_{B_{9,9}} & K_{B_{9,10}} & K_{B_{9,11}} & K_{B_{9,11}} \\
0 & 0 & K_{B_{10,3}} & K_{B_{10,4}} & K_{B_{10,5}} & K_{B_{10,6}} & 0 & 0 & K_{B_{10,9}} & K_{B_{10,10}} & K_{B_{10,11}} & K_{B_{10,11}} \\
0 & 0 & K_{B_{11,3}} & K_{B_{11,4}} & K_{B_{11,5}} & K_{B_{11,6}} & 0 & 0 & K_{B_{11,9}} & K_{B_{11,10}} & K_{B_{11,11}} & K_{B_{11,11}} \\
0 & 0 & K_{B_{12,3}} & K_{B_{12,4}} & K_{B_{12,5}} & K_{B_{12,6}} & 0 & 0 & K_{B_{12,9}} & K_{B_{12,10}} & K_{B_{12,11}} & K_{B_{12,11}}
\end{bmatrix} \tag{4.12}
$$

Now, to obtain the global stiffness matrix for the problem of Figure 4.3, we just compute the sum of $[k_A]$ and $[k_B]$ in their global form, thus:

$$[K] = [k_A] + [k_B] \tag{4.13}$$

4.2.8 Assembly of external forces vector

With the global stiffness matrix $[K]$ computed, the only thing we need to solve (4.1) is $\{P\}$, the vector of external forces. This vector has the purpose of measuring the contribution to each node of an external solicitation. Using the same problem as Figure 4.3, consider the following load case:

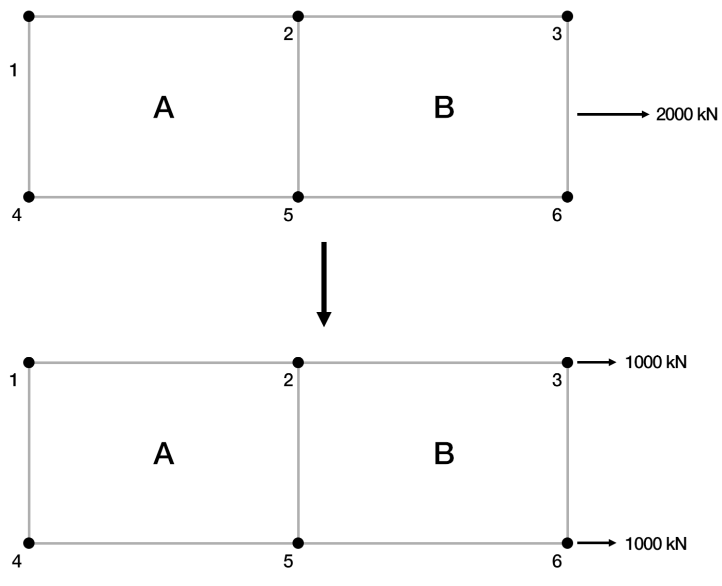

Figure 4.6: Representation of a problem discretized by two elements, A and B, and the effect of an external load on the surrounding nodes.

In this case, $\{P\}$ would be:

$$\{P\} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \\ P_{11} \\ P_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \\ 0 \end{Bmatrix} \tag{4.14}$$

Each P_i represents the force applied at DOF i. With this, we now have all we need to solve (4.1) and obtain the displacements.

4.2.9 Stresses determination

After solving (4.1) and obtaining the displacements at each node, we can obtain the stresses of each element. This is done with the following relations:

$$\left.\begin{aligned} \{\sigma\}_i &= [c]\{\epsilon\}_i \\ \{\epsilon\}_i &= [L]\{d\}_i \\ \{d\}_i &= \{u_i; v_i\} \end{aligned}\right\} \{\sigma\}_i = [c][L]\{d\}_i \tag{4.15}$$

Were $\{\sigma\}_i$ represents the stress, $\{\epsilon\}_i$ the strain and $\{d\}_i$ the displacements of each DOF at node i.

4.2.10 Solving problems with FEM

The concepts and examples with a dozen DOFs given until now are simple as to fully understand what is behind the custom method. Naturally, actual problems have hundreds and thousands of nodes, and twice as many DOFs. Solving a system of linear equations is trivial, but when there are thousands of equations speed becomes an issue, and methods to speed up the solving process need to be considered. Because of this, a method called LU decomposition [31] was used, which offers considerable speed increases when compared to more traditional methods.

LU decomposition consists in finding two matrices that, when multiplied together, result in a third matrix. For example, consider the matrix:

$$[A] = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 12 \\ 0 & 2 & -10 \end{bmatrix} \tag{4.16}$$

Now consider the following $[L]$ and $[U]$ matrices:

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}, [U] = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix}, \tag{4.17}$$

These $[L]$ and $[U]$ are obtained from $[A]$ through Gaussian elimination. We now have the following:

$$[L][U] = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 12 \\ 0 & 2 & -10 \end{bmatrix} = [A] \tag{4.18}$$

Thus it is seen that $[A]$ is actually the product of $[L]$ and $[U]$, where $[L]$ is a lower triangular matrix and $[U]$ an upper triangular matrix. What this means is that roughly half of each of these $[L]$ and $[U]$ matrices now are filled with zeros, and the zeros are distributed in such a way that greatly reduces the complexity of the system of equations, leading to faster solving of such systems.

4.3 Additional functionality

Until now, all concepts refer to standard FEM models, and the only advantage of implementing that kind of model is to speed up the process of obtaining stress distributions in an adhesive joint, which is a valid advantage that goes in line with the objectives of this thesis.

As we have full control of a custom model, it seems natural that we take advantage of that fact and build additional functionality that enriches the study of adhesive joints. The following functionalities were considered:

- Consider anisotropic adherends, through appropriate finite element formulation [15];
- Make it possible to analyse adhesives with varying properties through the overlap [13, 14];
- Take into account temperature considerations in the adhesive joint [25].

Chapter 5

Web application

5.1 Planned implementation

This thesis bases itself upon a MATLAB [32] created application *JointDesigner* (more details on this in Section 2.3.1). In reality, the idea in the beginning was to pick up the *JointDesigner* code and build upon it, implementing more features and making the application more complete and robust. However, after reviewing what was already done and based on the goals of this thesis, the original *JointDesigner* application was dropped due to some disadvantages.

The original *JointDesigner* was programmed in MATLAB, a decision made because of the robust language, the familiarity that students have with it (due to extended use) and the ability to create a user friendly interface. MATLAB also provided the option to compile the application as a standalone executable file, which was desired. Unfortunately there is a main issue with that MATLAB feature: to run, compiled applications need to have installed a MATLAB environment [32] in the target system (which is different from needing MATLAB itself installed, but inconvenient nonetheless). That would result in:

- Large files for installation — the MATLAB environment would need to be shipped together with the *JointDesigner* files, which would lead to file sizes in excess of 500MB, while the *JointDesigner* application itself would take little over 1MB;
- The MATLAB environment is a proprietary product, and requires a previous installation before using *JointDesigner*. This would not be professional, in the way that it would reveal to the user what language the product was developed in, and would also require two installations just to use one product.

Another reason is that, although MATLAB does provide tools to build user interfaces, those tools are not very complete and allow very little customisation (for example, at the time of writing this thesis there is not a control to include images, they have to be included by code through a plotting display, which is not a very user friendly way to display images). Because of all these reasons, it was decided to abandon MATLAB and develop the application in another language.

5.2 Equating other languages

Having discarded MATLAB, other languages were looked into to find the one most suitable for the task.

Starting with mathematics related software, MAPLE was evaluated but discarded due to lacking the ability to create a user interface and compiling applications. The only way MAPLE could be used would be to create a MAPLE spreadsheet and only users with MAPLE installed could run it. That is the opposite of what this thesis aims for (a straightforward and simple solution).

Having all mathematics related software that students are used to discarded, focus shifted to other more general languages more appropriate to create stand alone applications. Thus, Microsoft Visual Studio was evaluated, and although not mathematics oriented it proved to be a better option for a variety of reasons:

- Several languages to choose from, each with different levels of complexity (C, C++, C#, Visual Basic, etc.);

- Packages exist that could mimic mathematics features available in MATLAB and MAPLE, e.g. plotting suites, packages to solve systems of equations, etc;

- Complete freedom in the interface/application we could create, as a great number of commercial applications are written using Visual Studio.

The main disadvantage of all the previously stated languages was that the software created would run on Microsoft Windows, and although it is by far the most used operating system, it would not be ideal to limit the potential costumer base. The alternative would be to create more versions of the software to run on other operating systems, but there was no time to do that.

5.3 Reasons to build a web application

To overcome that big limitation, the decision was made to shift focus from a desktop application to a web application. It was decided to use the server-side language PHP for the code, which is a suitable general language and is supported by all web server hosting providers. The advantages now greatly outweigh the disadvantages, which are:

Advantages:

- Complete freedom to create the interface/application, limited only by the HTML code we provide to the users browser;
- Users with any operating system, any device at any place in the world can access the web application and use it without limitations — provided they have valid credentials;
- Vast amount of plugins/packages available that can provide assistance in the development of the product, e.g. plotting packages, navigation packages, etc.

Disadvantages:

- A web application runs on a web server that typically uses a Linux operating system [33]. This requires that we use some Linux compatible CAS to help with the more heavy calculations. MATLAB and MAPLE are obviously unsuitable, so another CAS software needs to be found that can fulfil these requirements.
- Any web server incurs server fees, fees that a desktop application would not be subjected to.

It was considered that the advantages outweigh the disadvantages considerably, and with careful work the disadvantages could be minimised.

5.4 Selecting a Linux based CAS

The main problem before starting to develop the web application is to select a Linux based CAS, as there are several methods that require heavy computing and PHP only provides basic mathematics functions. Simpler methods that require substituting values in single equations can be implemented in pure PHP, but when the method requires that a system of linear or differential equations is solved we need a CAS to assist in that task.

It should be noted that, as the CAS software will carry out the calculations, there is the possibility to use more than one software. Consider the following example: if CAS software

X provides faster results for matrix operations and CAS software Y provides quicker equation solving procedures both can be used, so we'll effectively use only the stronger points of each software.

5.4.1 Brief technical background

Computers run only one type of code, called machine code, which is a highly optimised set of instructions and operations performed by the CPU (Central Processing Unit, a component of a computer). Therefore, all programs that run on a computer must run in machine code, which is a special type of code hidden to the user and the programmer that is automatically created just for the purpose of running on the CPU. There are a variety of programming languages (C++, Visual Basic, Pascal, Fortran, etc.), but every one of them must in the end be translated to machine code.

This translation to machine code depends on how the software was created:

- When the software is created in languages like C++ or Fortran, it is in the end compiled to machine code. This effectively means that the person who wrote the program presses a button, and all the lines of code written by the programmer are translated into machine code and into an executable file. When that executable file is opened, all there exists in it is machine code which then runs directly on the computer, meaning that the executable can be sent to any computer and will run very fast, because it is already in the language the CPU understands. Therefore, languages like C++ and Fortran are called **compiled languages**, meaning they compile directly into CPU instructions and produce an executable file for the end user to run;

- Other languages exist, e.g. Ruby and Python, that differ from the previously mentioned languages. These languages do not compile into an executable file, they are instead sent to the end user without being optimised for the CPU first, meaning they must be translated to machine code when the user tries to run them. This means that when the user opens the program, the first step is translating the code into machine code, and the second step is running the machine code. These types of languages are called **interpreted languages**.

- There are also languages that are both, compiled and interpreted, so they are first partially translated, and the remaining non-translated part must be translated by the end users computer, albeit the translation that need to be done is significantly less than in the case of purely interpreted languages.

In short, a compiled language gives the end user the ability to **run the executable directly**, while an interpreted language must **first be translated into machine code**. It is therefore understandable that compiled languages result in faster running programs, because interpreted languages need more time for the translation of the code into machine code.

5.4.2 Singular

Singular was the first CAS looked into that appeared to fulfil the needs of our project. Its definition [34]:

> "**Singular** is a computer algebra system for polynomial computations, with special emphasis on commutative and non-commutative algebra, algebraic geometry, and singularity theory. It is free and open-source. Singular provides highly efficient core algorithms and a multitude of advanced algorithms in the above fields."

Singular is written in C++, a compiled language, making it very fast. Besides being a compiled language, the fact that it provides highly efficient mathematical functions makes it very attractive for our needs.

5.4.3 Maxima

Maxima was another option considered, and is one of the most widely used CAS solutions available. By definition [35]:

> "**Maxima** is a system for the manipulation of symbolic and numerical expressions, including differentiation, integration, Taylor series, Laplace transforms, ordinary differential equations, systems of linear equations, polynomials, and sets, lists, vectors, matrices, and tensors. Maxima yields high precision numeric results by using exact fractions, arbitrary precision integers, and variable precision floating point numbers."

Maxima is written is Lisp, a language that is both compiled and interpreted. This should make it slightly slower than Singular, but it also provides stronger and wider support of mathematical and algebra concepts, e.g. it natively solves systems of differential equations, while Singular does not.

5.4.4 Sage

Sage was the third possibility found, and defines itself as [36]:

> "**Sage** is a free open-source mathematics software system licensed under the GPL. It combines the power of many existing open-source packages into a common Python-based interface. *Mission: Creating a viable free open source alternative to Magma, MAPLE, Mathematica and MATLAB.*"

Sage is written in Python, which is an interpreted language. This effectively means that it should be the slowest CAS analysed until now, but has other advantages that make it worth to consider: it incorporates the Maxima solution mentioned previously and several other functions, making it the most complete CAS of all three.

5.5 Testing the CAS solutions

Having selected the previous possible CAS suites, it is now necessary to test each CAS to see how it performs solving the problems we will face.

5.5.1 Hart Smith test

This test was chosen because it requires solving a system of 3 linear equations for 3 variables, a simple and necessary procedure to implement some analytical methods. The equations are the following:

$$\frac{P}{l\tau_p}(\lambda' l) = 2\lambda'\left(\frac{l-d}{2}\right) + (1-K)(\lambda' d) + K\tanh(\lambda' d) \tag{5.1}$$

$$\frac{P}{\tau_p}\lambda^2\left[1 + 3k(1-\nu^2)\left(1+\frac{t_a}{t}\right)\right] = 2\left(\frac{\gamma_p}{\gamma_e}\right) + K\left[2\lambda'\left(\frac{l-d}{2}\right)\right]^2 \tag{5.2}$$

$$2\left(\frac{\gamma_p}{\gamma_e}\right) = K\left\{\left[2\lambda'\left(\frac{l-d}{2}\right) + \tanh(\lambda' d)\right]^2 - \tanh(\lambda' d)\right\} \tag{5.3}$$

These equations are part of the analytical method Hart Smith, already presented in the previous chapter. This system should be solved for variables P, K and d, while the rest are given constants. k is a function of P, so at the start of the analysis k is set to an arbitrary value k_s, and after solving the system a new k_f is found (that depends on the value of P). The system of equations is then iteratively solved until $|k_f - k_s| < e$, where e is the value of the error we define as acceptable.

The test was performed based on the following joint (Figure 5.1):

$$E_1 = E_2 = 210GPa \qquad \nu_1 = \nu_2 = 0.3 \qquad E_a = 3GPa \qquad \nu_a = 0.35$$

$$\tau_p = 30MPa \qquad \gamma_e = 0.027 \qquad \gamma_p = 0.05$$

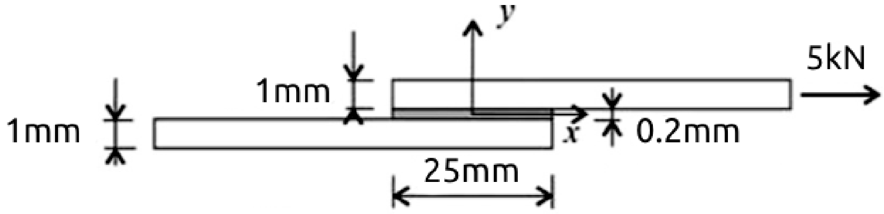

Figure 5.1: Joint Geometry used for the Hart Smith test

The remaining variables of (5.1), (5.2) and (5.3) are given elsewhere [23]. The value of the error, e, assumed for this tests purposes was $e = 0,001$, and an initial value of $k_s = 0$ was given. After running the same problem in the different CAS solutions, the results were the following:

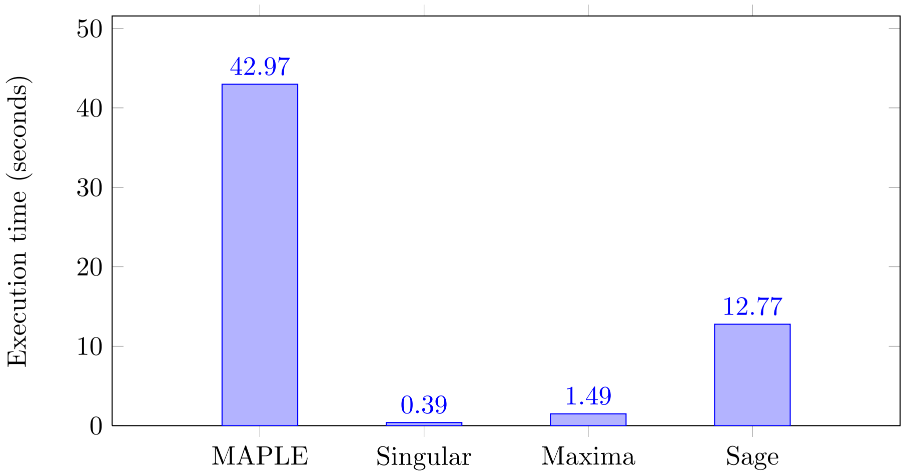

Figure 5.2: Total time that each of the tested CAS took to run the simulation.

Each software performed 3 iterations, effectively solving the mentioned system of equations 3 times. It is clearly visible that Singular outperforms every other CAS, followed by Maxima, Sage and lastly MAPLE.

MAPLE is provided as a comparison basis, and all three CAS solutions performed as expected comparatively with each other. Quantitatively, Singular shows a significant difference in speed from the others, making it almost 4 times faster than Maxima, and 32 times faster than Sage.

Because of these test results, we decided to adopt both Singular (because of its speed) and Maxima (because of the extra functionality it provides) to be used when needed, together with the JointDesigner web application.

5.6 Front end

The front end is what the user sees: the web site layout, logo, buttons, etc. For the front end, the following decisions were made:

General look: It was decided that the site would be build using a two-column layout (Figure 5.3). This allows us to use the main column to display all necessary information and results, and the sidebar serves various secondary functions:

- On an analysis, the sidebar keeps a summary of all the analysis values (geometry, material, analytical method, etc.), and also makes it possible to quickly change each of the values when viewing results. This makes it much faster and easier for the user to change some property and instantly update the results;
- When first accessing the web application, the sidebar functions as a login form, so the user can enter his credentials and start using the site;
- When on the remaining pages (materials database, loading an analysis, choosing what analysis to study, etc.) the sidebar shows some information regarding the user (his name, institution, and when the last login occurred).

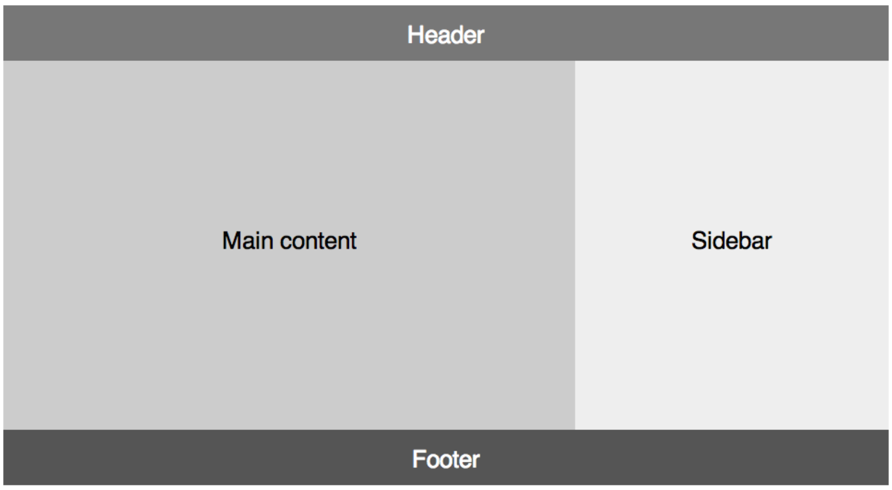

Figure 5.3: Definition of a 2 column layout: one column hosts the main content, and a smaller column (the sidebar) hosts secondary information.

Logo: The logo should follow the idea of the remaining site: simple and minimal look, so that the user can focus on what matters (joint analysis) and still be confident around the site (after all, a carefully created interface inspires much more confidence). With that in mind, the logo was designed based on the logo of the previous version of JointDesigner, as Figure 5.4 shows.

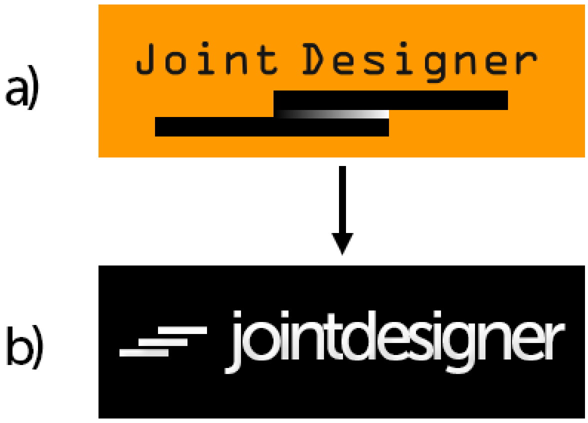

Figure 5.4: Evolution of the logo of the web application: a) the logo of the MATLAB version of JointDesigner; b) the currently used logo.

Main content: The main content, as the name states, is the main area of the application, as such it must be uncluttered and simple. Lots of details were put into the main content area, some of those are:

- When choosing the properties for the first adherend, both adherends adopt those properties, thus saving time. This is because in most situations both adherends are of the same material, and in case they are not, the second adherend can be easily changed as well;
- When inputting properties in an analysis, a hint accompanies the user so that he never forgets what the input box he is using is actually for;
- All buttons/input boxed/headers maintain the same look throughout the site, so that there is no confusing about each element's function;
- When calculations are being done, the user is warned that the results are being computed, and when they are ready the warning automatically disappears. This is so that the user does not think something went wrong (disconnected from internet, application crashed, etc.).

Sidebar: The sidebar's purpose, as said, is to summarise the analysis, between other functions. The layout was structured so that every detail about the joint is clear, and icons exist to tell the user where to click if he desires to change a value.

5.7 Back end

The back end is what happens behind the scenes, what the user does not see but must happen for things to work. On the back end various things exist: the analytical methods used to compute results, the functions to interact with the databases (save analysis, add/remove from materials database, authenticate users, etc.), all the code to handle generating exported files (Excel, PDF, etc.), and several other smaller details.

Analytical methods: By far the most important thing on the backend are the analytical methods, without which this application would not exist. They are built around a modular interface (Figure 5.5): a "container" holds all the information about the joint, and then depending on the user request it retrieves the module of the appropriate analytical method. This makes it very easy to develop new methods in the future, and the container must only be updated to be made aware of the new module's existence, while everything else updates accordingly and works automatically.

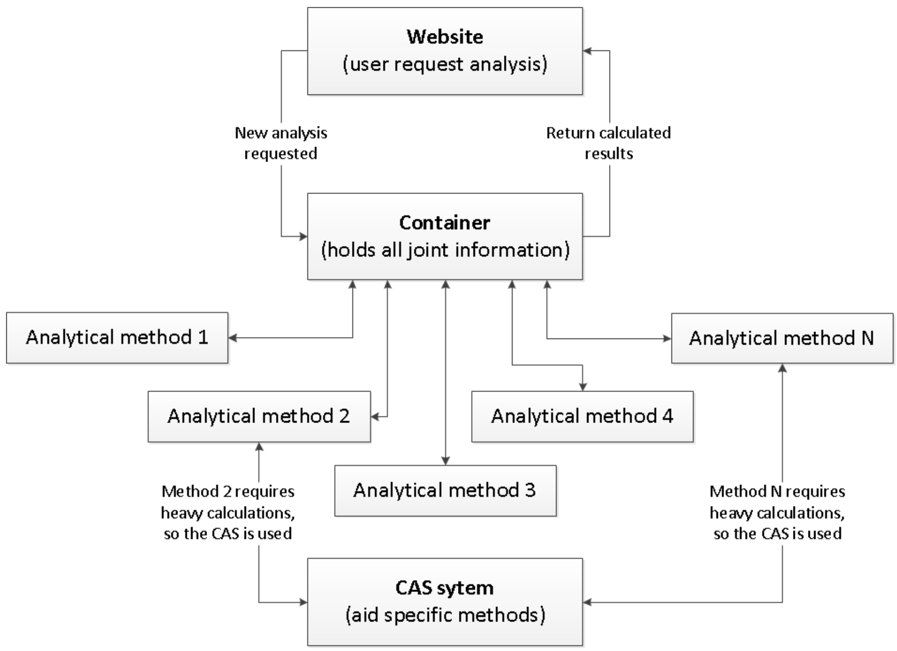

Figure 5.5: Representation of the modular interface for the analytical methods: the user requests an analysis, the container holds all joint information, then calls the appropriate module, which calculates the results (with the help or not of the CAS) and sends back the calculated values to the container, which then puts everything in order to send back to the website.

Export utility: The export utility makes it easy for the user to save/download files with the results calculated by the web application. He can then use the data further, send the files, etc. The three exportable filetypes supported are:

- Excel: the user can open the results in Excel, make additional changes, create plots from the data, share the files, etc.;
- CSV (Comma Separated Values): it is a standard type of file for numerical results, and is supported by a wide array of applications, including Excel;
- PDF: a PDF report with the calculated results can be exported, making it easy to print or share the files.

Print utility: The user can print directly from the results page (without having to download a file) by simply clicking the "Print" button. This will print all the resulting graphs.

Save analysis: In case the user wishes to keep the values calculated available for later use, he can save the analysis. It is saved to the user's personal space, i.e. only available to him, and can be loaded again any time in the future.

5.8 Administration

There is something else that also needs to be built to allow us to manage the web application: an administration interface (Figure 5.6). It should accomplish tasks like:

- Add and remove users with access to JointDesigner;
- Add and remove standard adhesives and adherends.

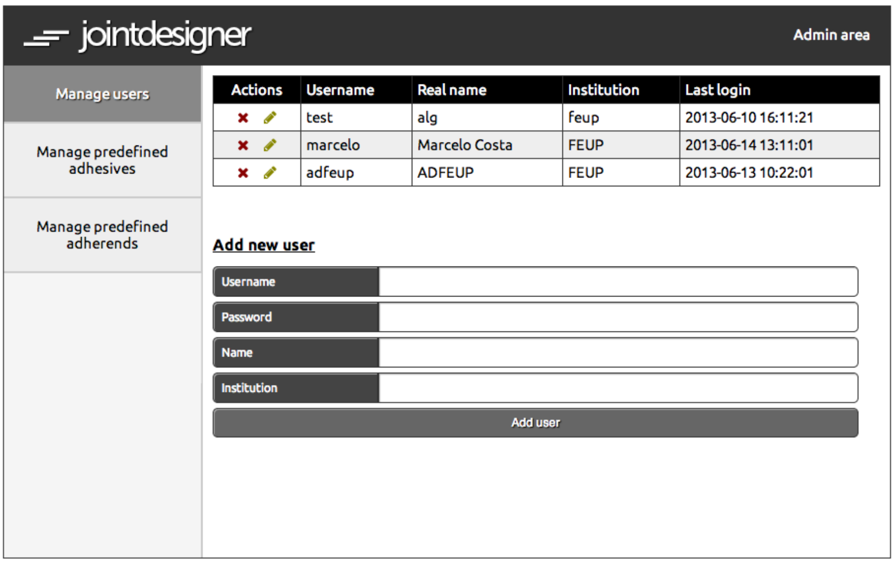

Figure 5.6: Administration interface built for JointDesigner.

5.9 Final solution

The adopted solution works in the following way:

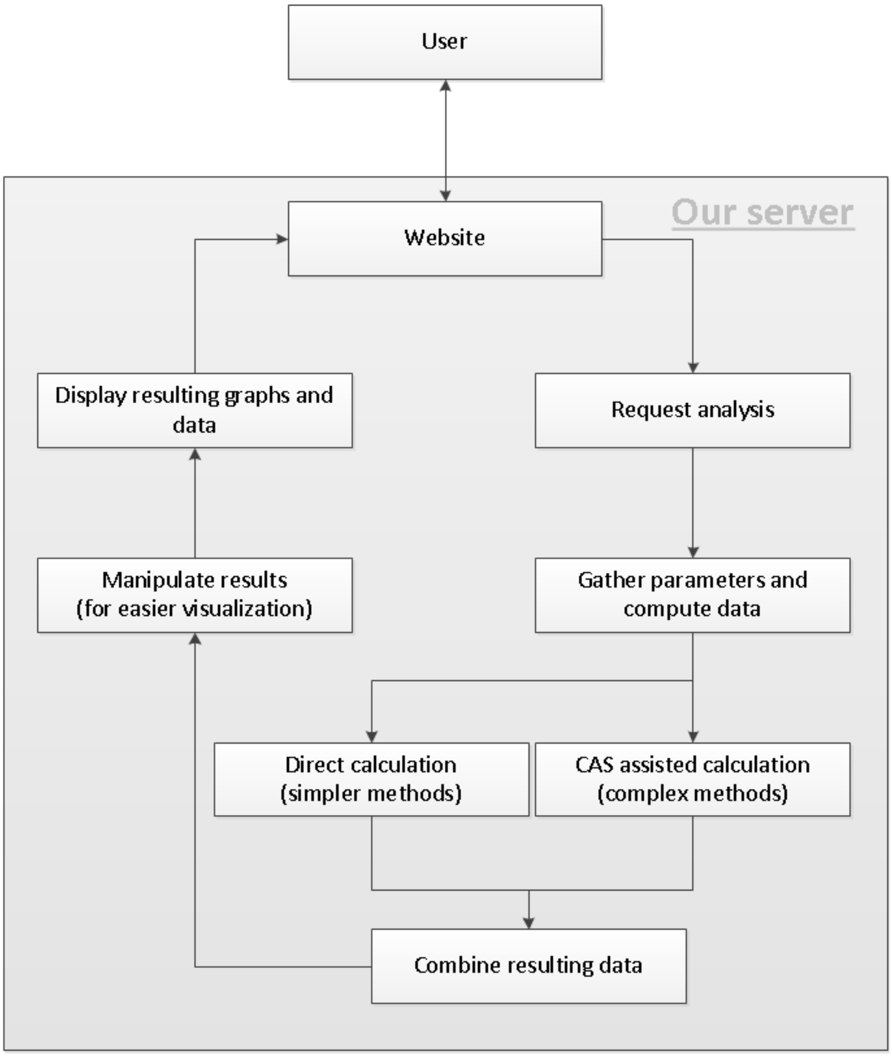

Figure 5.7: Web application inner working

5.10 Final web site

In this section the pages the user comes in contact with are presented. When accessing the JointDesigner's domain, the user is welcomed with the page shown in Figure 5.8.

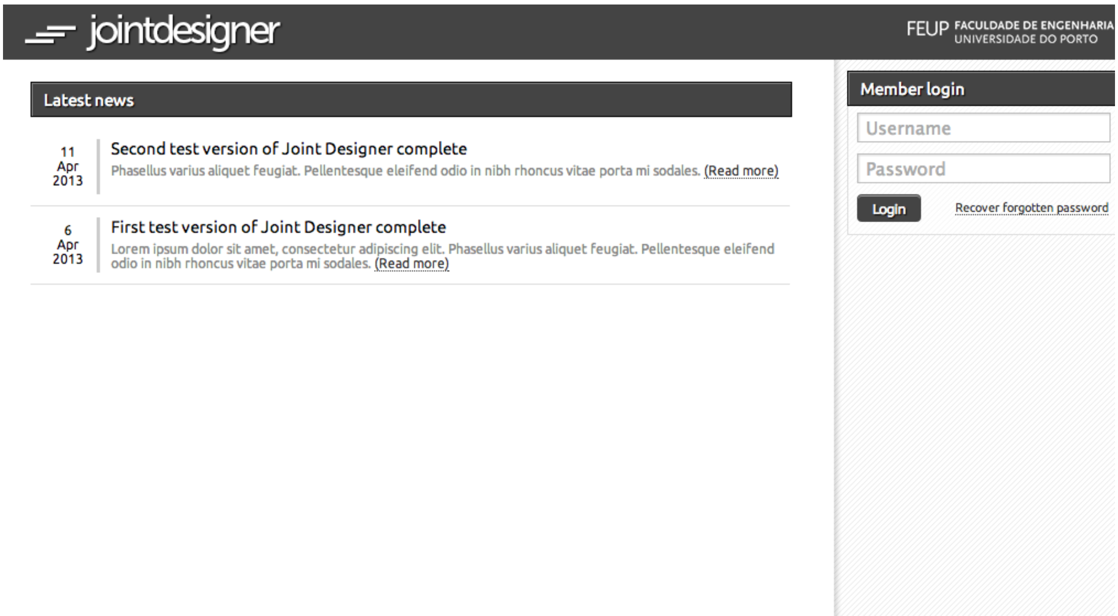

Figure 5.8: Front page.

After using the sidebar at the right to login, the user is then taken to the screen pictured in Figure 5.9.

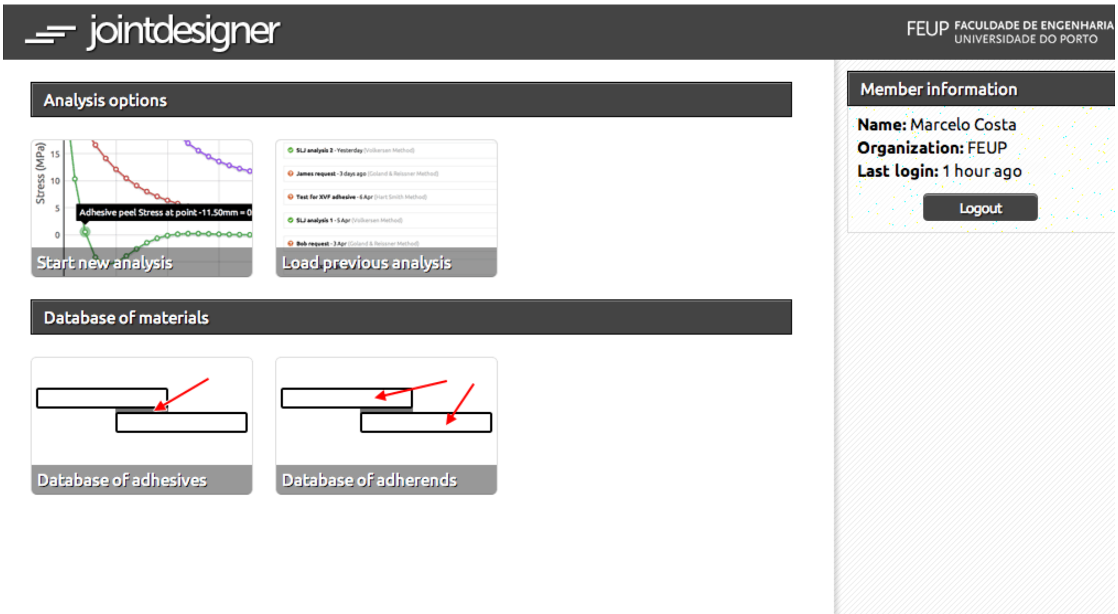

Figure 5.9: Start page.

What is visible in Figure 5.9 is the main page of JointDesigner, the place where the user can choose what to do. He has the following options:

- Start a new analysis;
- Load a previously saved analysis;
- View and change the database of materials (both adhesives and adherends).

Starting with the database of adhesives, the page available for it is shown in Figure 5.10.

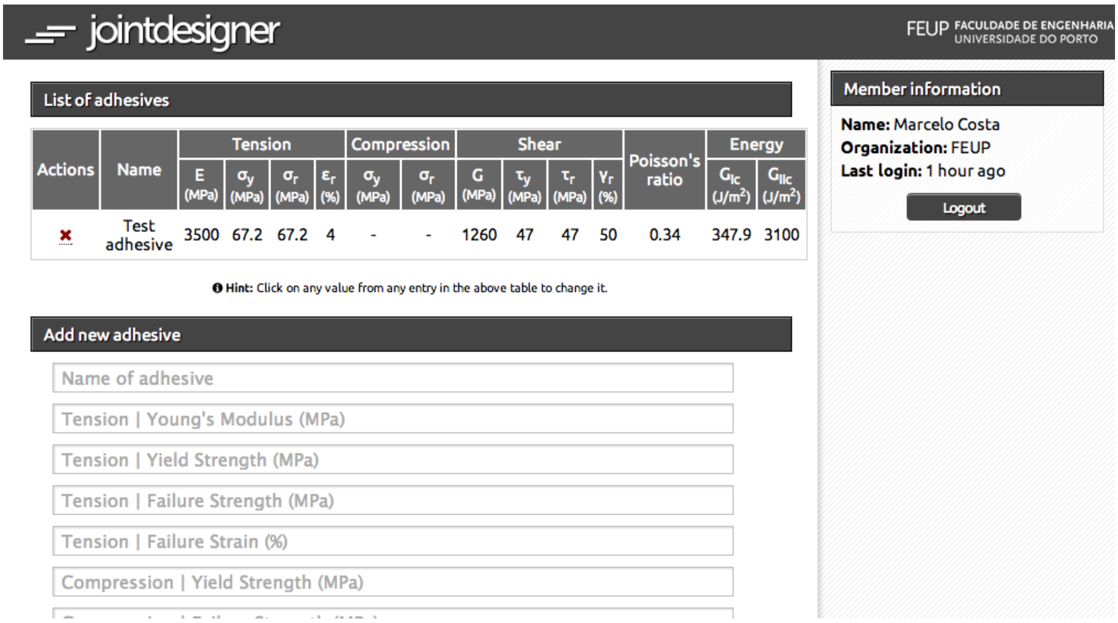

Figure 5.10: Materials database.

The following functions are available at this page:

- The user can add custom materials. To do this, a form on the bottom half of the page exists with the title "Add new adhesive", where the various properties about the adhesive can be inserted. Not all values have to be inserted and most fields can be left blank;
- There is a table on the top of the page where the user can visualise existing custom materials;
- To edit any parameters of each adhesive, the user must only click on it. Then, a custom popup asks the user what the new value for the clicked property is, and upon confirming it the value is automatically updated on the database.

As can be seen in Figure 5.9, besides the Database of Adhesives (shown on Figure 5.10) there is also a Database of Adherends. The page is mostly the same as the one shown on Figure 5.10 and also works similarly, adding custom adherends to the database.

Now that the database of materials functionality is shown, let's assume the user clicks the button "Start new analysis" (Figure 5.9). In that case, the page presented in Figure 5.11 is shown.

Figure 5.11: Starting new analysis.

Only SLJ analysis is available in this first version. Upon clicking the "Single Lap Joint" button, the next page is the one shown in Figure 5.12.

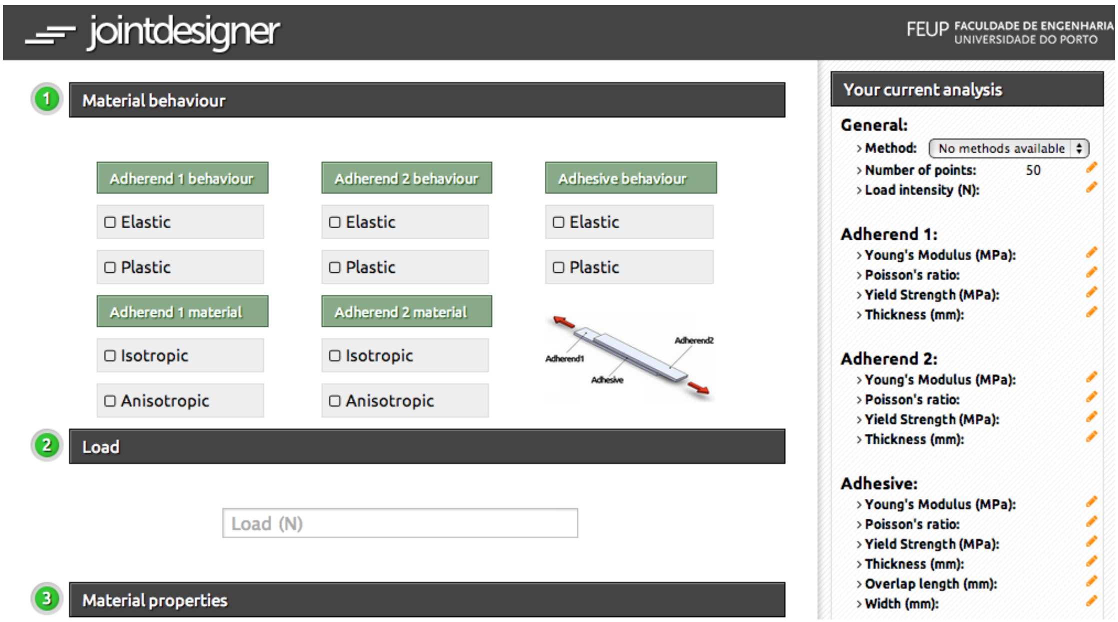

Figure 5.12: New single lap joint analysis.

This can be considered the main page of the site, as it is where the actual analysis can start to be computed. Some things should be noted:

- The sidebar on the right hosts all the main properties regarding the analysis, including the adherends and adhesive geometry and materials, as well as the method chosen for the analysis. It is relevant to mention that the method selection in the sidebar says "No methods available", and that is because the methods are automatically chosen only after defining the "Material Behaviour" section;
- The rest of the site, besides the sidebar, consists on a series of numbered steps: Material behaviour, Load, Material properties, Material geometry, Analytical methods and Failure criterion.

After defining the details on the "Material behaviour" section, the following happens:

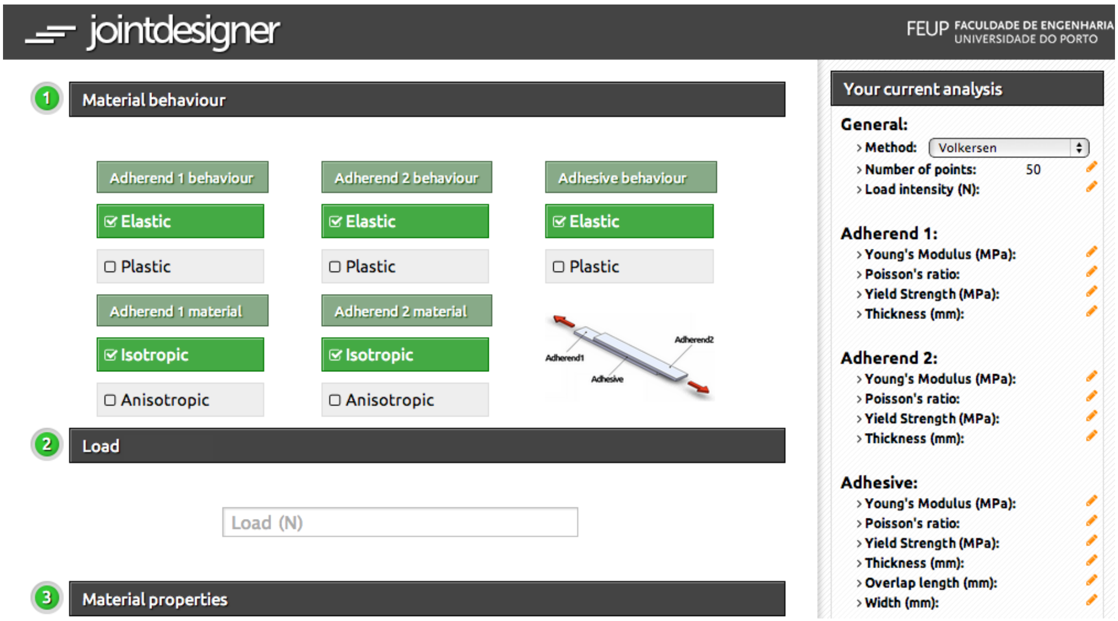

Figure 5.13: New single lap joint analysis, after defining the material behaviour.

Now that all the material behaviour is defined, the site can choose what analytical methods the user can pick. On the sidebar, the "Method" has now changed and shows the default Volkersen's method. If the user clicks that dropdown, a list of more methods is available.

Figure 5.14 and Figure 5.15 show intermediate steps of the analysis page.

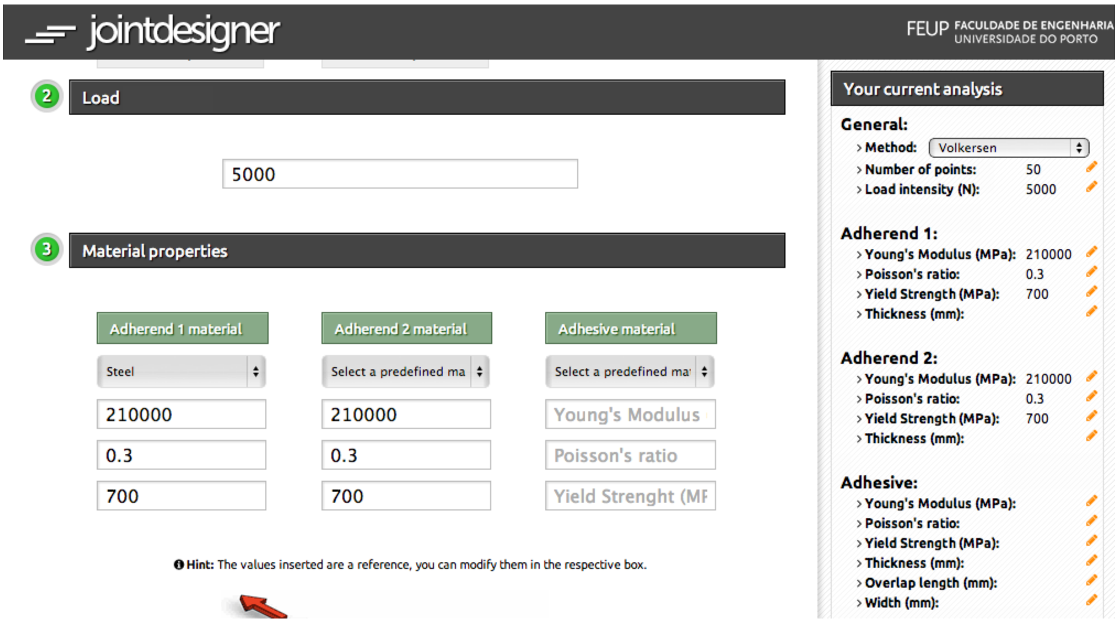

Figure 5.14: New single lap joint analysis, after defining the applied load and some material properties.

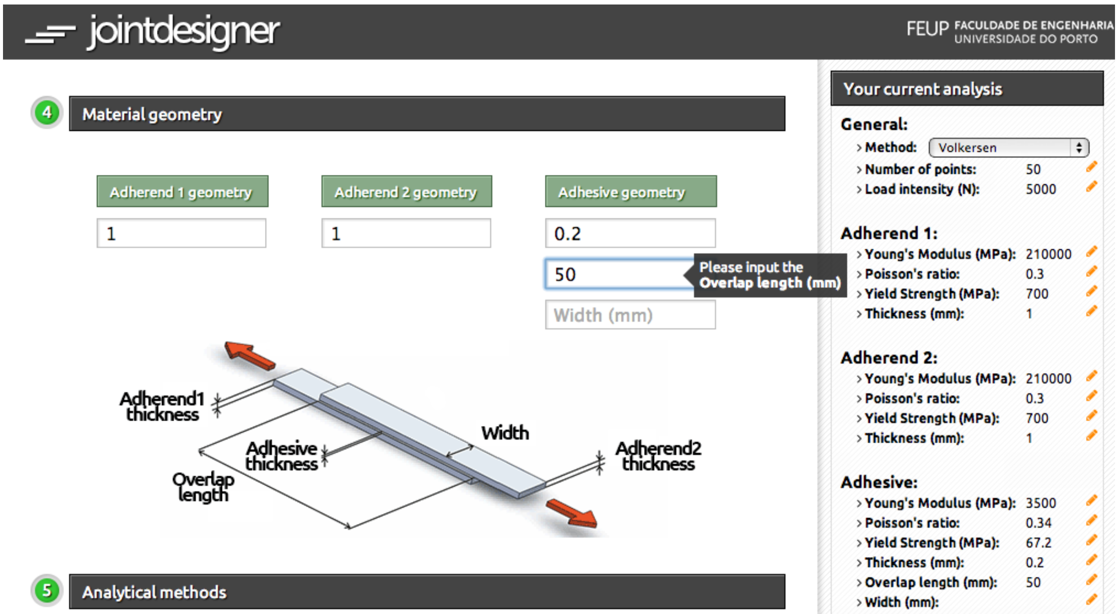

Figure 5.15: New single lap joint analysis, after defining the adherends and adhesive thickness.

Next is the step to select the analytical method. A button for each available method is available, and the user chooses the desired one. Figure 5.16 shows the Goland & Reissner's method as the chosen one.

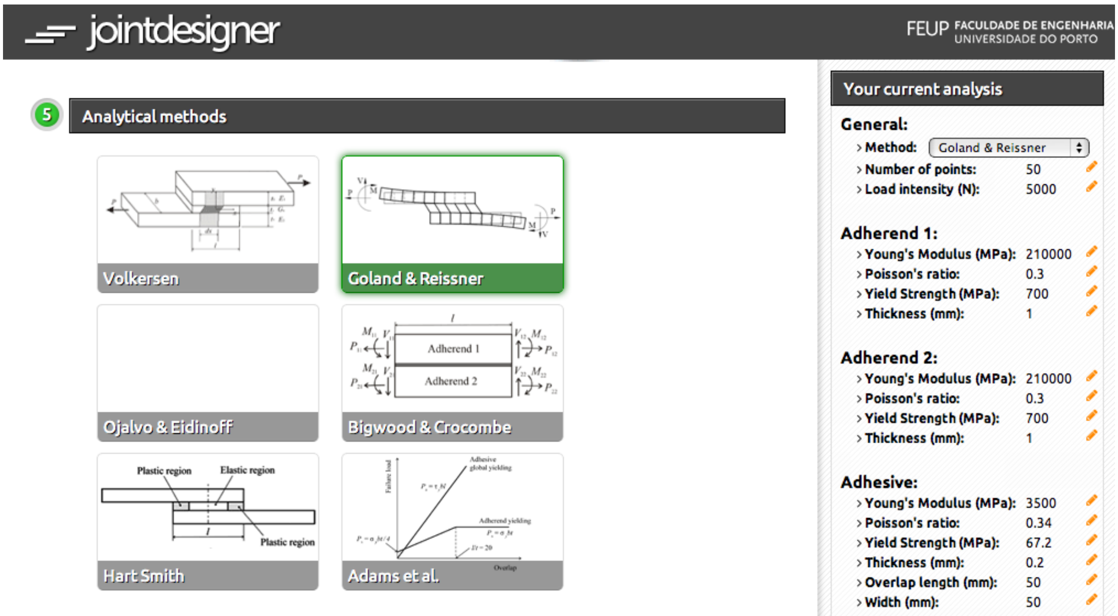

Figure 5.16: New single lap joint analysis, chosing the analytical method.

Finally, we can define the "Failure criterion", a step that depends on the selected method (Figure 5.17). As we chose Goland & Reissner, which provides peel and shear analysis, we can define the failure criterion for each of the stresses here, although it is not required.

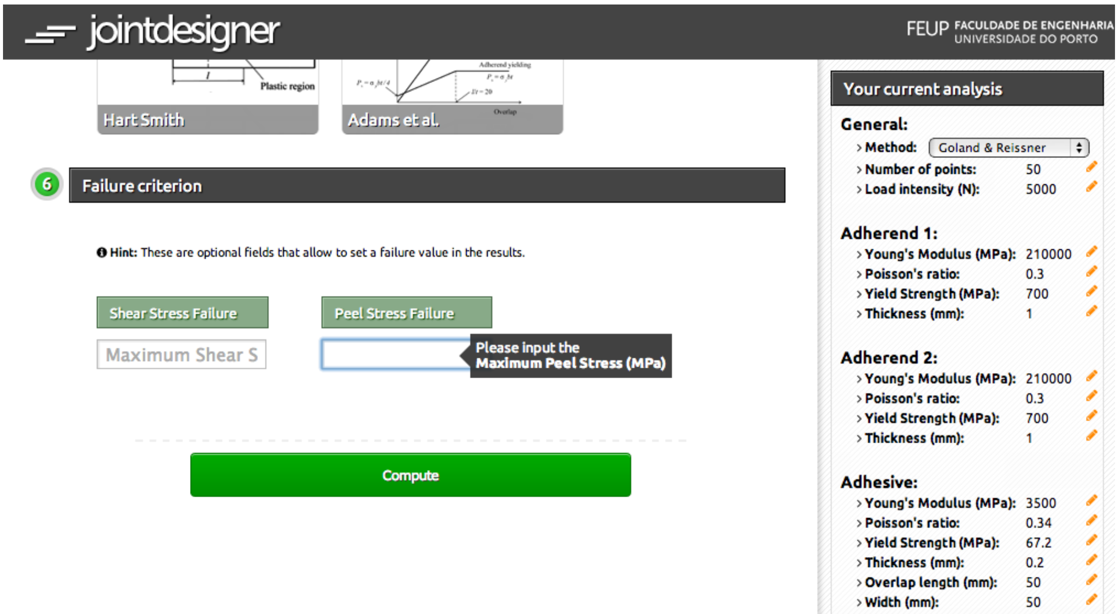

Figure 5.17: New single lap joint analysis, defining the failure criterion.

Then after we use the "Compute" button, the results appear, as represented in Figure 5.18.

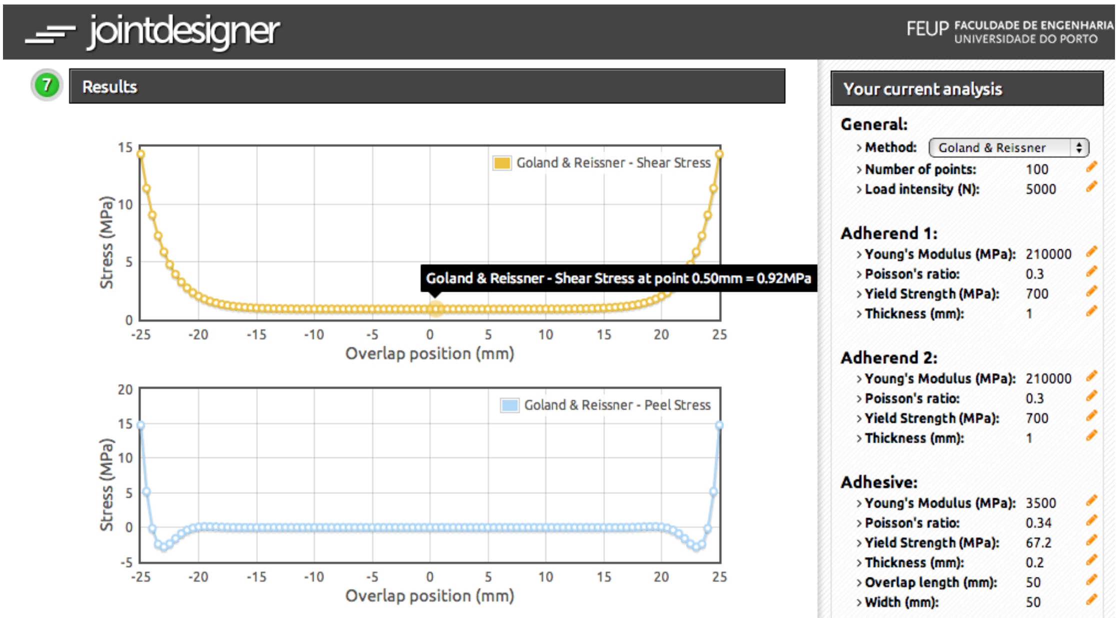

Figure 5.18: New single lap joint analysis, resulting plots.

Several important things should be noted regarding Figure 5.18:

- All the resulting plots are separated. We could put both the shear and peel results in the same plot (as in Figure 3.3 c)), but we could not represent the failure criterion like that (both failure criterions would be in the same plot, and it could be confusing). It is also easier to visualise each result separately, so the decision was made to separate all plots;
- We can view the numerical results in each point of the overlap by simply hovering the mouse over the desired point, where a tooltip appears with the information, as shown Figure 5.18;
- The sidebar can be used to change values of the analysis directly. For example, we can change the adhesive's Young's modulus by clicking on its value on the sidebar (3500) or in the orange pencil next to it. This causes a popup to appear, asking for the new value for that parameter, and upon confirming the new value the resulting plots are automatically recalculated and displayed to the user. This saves a lot of time, eliminating the need to go back, change the value, and then use the compute button again.

After the plots are displayed, export options are also available at the bottom of the same page, as Figure 5.19 shows.

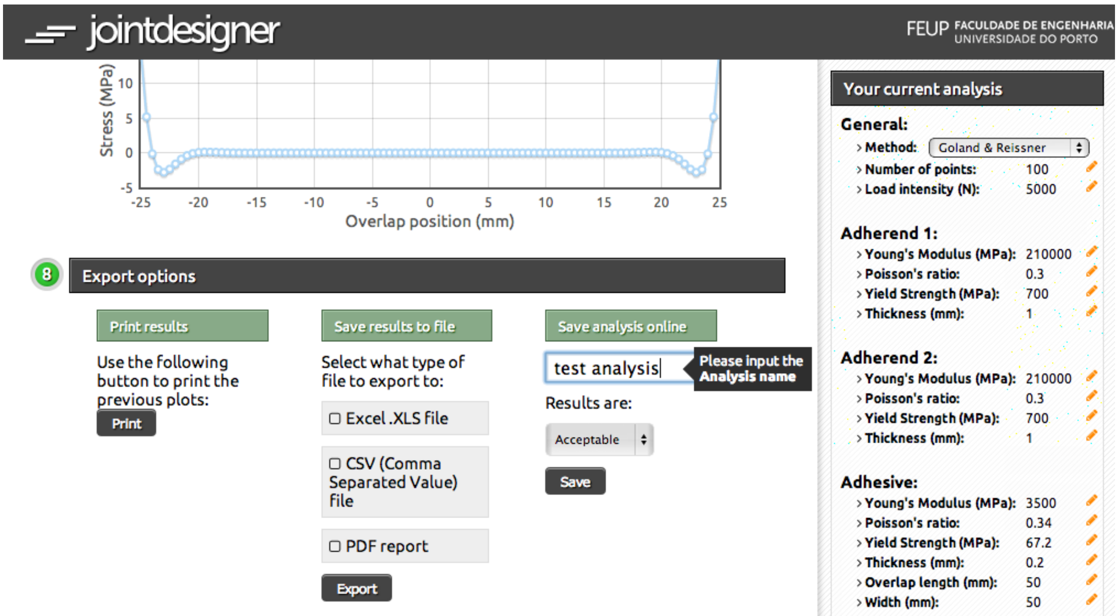

Figure 5.19: New single lap joint analysis, export options.

The "Print" button triggers a popup that automatically sends the current plots to the printer, the "Export" section creates the appropriate files and triggers a download by the user, and the "Save analysis online" section allows the user to save the results in their private area, so they can later access them. After clicking the "Save" button, the user is redirected to the same page, as shown on Figure 5.9. There, if we use the option "Load previous analysis", the page shown on Figure 5.20 appears.

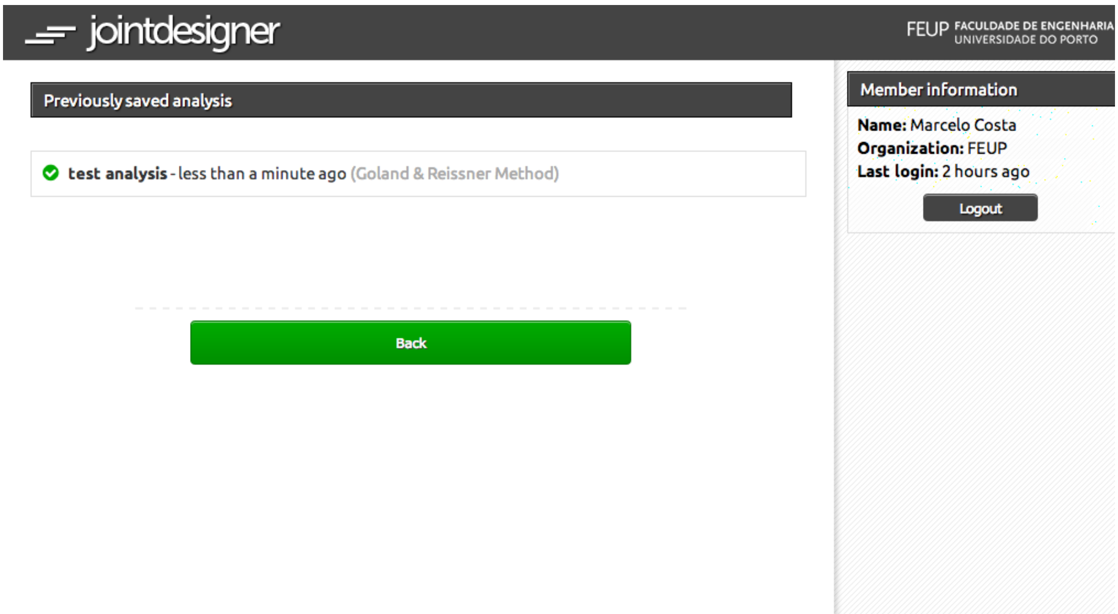

Figure 5.20: Loading a previously saved analysis.

Clicking on the desired analysis name we are taken to the same page as Figure 5.12, but with all the values automatically filled, as Figure 5.21 shows.

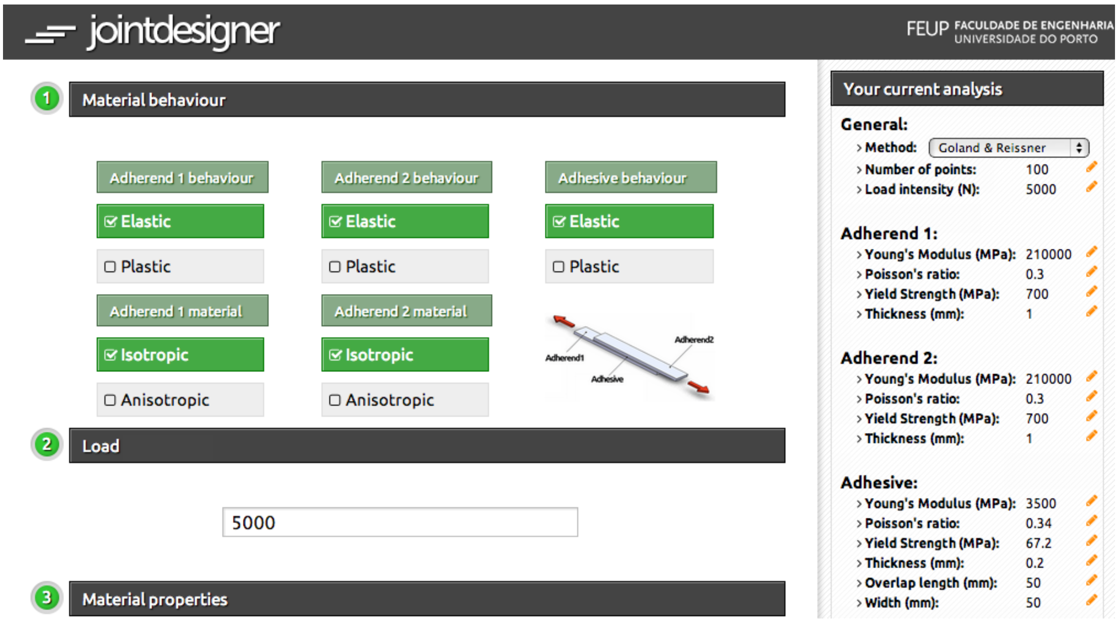

Figure 5.21: Loading a previously saved analysis, with the new analysis page filled with the saved analysis' values.

Chapter 6

Conclusions

After the development of this web application, the following can be concluded:

- All the main objectives that were initially set and some secondary ones were implemented with success:
  - A web application that effectively computes results using several analytical methods and is easy to use was fully created and tested;
  - Analytical methods for SLJ in various situations were implemented, as seen on Table 3.2;
  - Secondary goals, like saving results online and exporting results for them to be used in other applications, were also implemented and are working;
  - Sharing of analysis results directly with coworkers was not implemented due to time restrains.
- The web application is ready to be deployed and used by engineers, seeing as it is fully tested and the analytical methods implemented have been verified both historically (their usage is well documented in the scientific community) and numerically (the results obtained with the web application have been compared with MAPLE implementations of the methods).

In the end it is believed that the produced work is a valuable tool for an engineer that designs adhesive joints to possess, as it is very easy to use and produces valid results that can be exported and then used as the engineer wishes.

Chapter 7

Future work

Due to the enormous potential that an application of this type has, the only big limitation to implement new features is time. That has certainly been the case with JointDesigner, and although a lot has been accomplished more things were planned that could not be implemented in time. Some of those are:

- Implement methods for other joint types besides single lap joints: double lap joints, tubular joints, corner joints, T-joints, etc.;
- Develop the custom FEM method further, adding things like: composite adherends, temperature considerations, 3D analysis, etc.;
- Provide ways to share analysis results with coworkers: sharing results directly within the site, sending of e-mails with results directly from the site, etc.;
- More features for the web application would be useful: a user control panel (to change their name/password/etc.);
- Have the web application commercially available, because due to some bureaucratic delays that could not be accomplished in time;
- Provide custom functionality to clients that request it, allowing for extra monetizing possibilities.

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